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Song, Hui S.

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HUI S. SONG

UNITED STATES
NAVAL POSTGRADUATE SCHOOL

DEPARTMENT OF THE NAVY ★ NAVAL POSTGRADUATE SCHOOL ★

THESIS

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Hui S. Song

Lieutenant, Republic of Korea Navy

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ABSTRACT

Simulation techniques for nonlinearities in servomechanisms using the analog computer have long been subjects for investigations. In laboratory, these techniques play important roles for prediction of performance or in the study of nonlinearities in servomechanisms.

Many successful methods in this field were developed already. Seven different basic nonlinearities have been chosen for investigation in this report. Net works for simulation of these nonlinearities are then generalized or initiated with complete analysis. Method of scaling is discussed, and illustrative simulation in a second order system was included to illustrate the ease of simulation with reasonable accuracy.

The writer wishes to express his appreciation for the assistance and encouragement given him by Professor George Julius Thaler of the U. S. Naval Postgraduate School in this investigation.

SYMBOLS

$\dot{\theta}$	Output velocity
θ	Output position
ε	Error signal
r_f	Diode forward resistance
r_b	Diode backward resistance
Δ	Amount of backlash
C	Amount of coulomb friction
D	Amount of dead space
$\frac{p}{2}$	"pull in" voltage of relay
$\frac{d}{2}$	"drop out" voltage of relay
$\overline{\theta}$	Bar on top indicates computer value
M	$\frac{\theta_o}{\theta_i}$
M_{pt}	Maximum M for transient response
$M_{p\omega}$	Maximum M for frequency response
ζ	System damping factor
$\theta_{(p)}$	Subscript p denotes problem value
$\theta_{(c)}$	Subscript c denotes computer value

$$\alpha\theta \triangleq \frac{\theta_{(p)}}{\theta_{(c)}}$$

$$\alpha t \triangleq \frac{t_{(c)}}{t_{(p)}}$$

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1. Introduction.

In most existing physical systems, it is usual to have some degree of nonlinearity in the system, however small the degree may be. Or the system may deliberately be designed nonlinear such as a relay system utilizing the available torque to it's full advantage.

If this is the case, equations describing the existing physical system becomes nonlinear, the concept of roots of a characteristic equation is not defined and there is no formal mathematical relation between the time domain and the frequency domain, since the principle of superposition does not apply, thus invalidating the basic tools for the analysis of linear system. For the transient response of the system, numbers to predict transient performance are not readily available.

Thus a method of prediction of performance for nonlinear systems becomes very important in the field of design of servomechanisms and to understand the effect of nonlinearities which may be present in the system; or in the design of nonlinear systems such as relay servo.

Todate, the method of analysis of nonlinear systems is largely confined to phase plane method. But we are all aware of the laborious works and time involved with this technique as the order of differential equation describing the system becomes higher. And the error involved in the evaluation of transient performance from phase plane method in higher order system may become greater.

One method of predicting system transient performance and frequency response directly at the laboratory is to simulate the problem to obtain solutions of equations that describe the system by use of analog computer.

Although a number of basic circuit to simulate nonlinearity have been introduced, a general tendency to avoid utilizing such circuits still remains. This report is dedicated to encouraging the method of analog computer simulation in the study of the behavior of nonlinear mechanism which are commonly met in the usual servomechanism, by forming up the circuits, analyzing them in detail and by establishing scaling.

These circuits are formed such that it will require the minimum possible number of amplifiers with reasonable accuracy for engineering purposes. Seven basic nonlinearities are selected for the subject of investigation in the report, and the method of simulation is based on the transfer function method to minimize the number of amplifiers and for ease of scaling for all nonlinearities selected. Section 2 contains separated circuits to simulate basic nonlinearities with detailed mathematical analysis and general scaling. Section 3 illustrates actual usage of the cirucits discussed in section 2, in a second order system with resultant transient, phase plane and frequency response curves. The method of scaling is illustrated in sample simulation in section 3.

2. Circuits for simulation of basic nonlinearities.

In this section, basic nonlinearities encountered in the usual servomechanism are represented by simulation circuits for use with analog computer. These circuits are analyzed and simulated in the Donner analog computer. Statistic curves were taken and attached to this report.

Scaling in general....For scaling of a system with nonlinearity, it is best to neglect the presence of nonlinearity and calculate maximum values of the desired quantities to establish a proper scaling for the system. In this way, the main circuit to represent the linear portion of the system can be formed, after which the portion of the nonlinear circuit can be inserted. If the availability of components necessary for the nonlinear simulation circuit is limited, such as voltages of batteries or potentiometer size, it may become desirable to select scaling factors convenient for the formation of the nonlinear portion of the circuit. In that case, it would not require a great deal of time and effort to change scaling factors once established.

The nonlinearity is then inserted in the linear circuit to form the main branch of the simulation circuit as in the case of backlash, relays and dead space etc., or added to the linear circuit to form a compensatory additional circuit of the system as in the case of coulomb friction.

If the nonlinear circuit is inserted into the linear circuit to become a part of the main branch, the scaling of the nonlinear circuit must be such that the scaling factors of the quantity be identical with that of the linear circuit at inlet and outlet stations.

If the nonlinear circuit is added to the main linear circuit to become a compensatory additional circuit, scaling must be done in accordance with the equation describing the portion of the main branch to which the nonlinear circuit was added.

These are covered in detail in section 3 of sample simulation of a second order nonlinear system.

A. Saturation.

(a) Saturation phenomena in amplifiers etc., is represented by input vs. output relation as shown in (Fig. 1-a).

(b) Circuit to represent saturation.

i. Circuit utilizing batteries (Fig. 1-b).

Where rd1, rd2 are diode resistances, which further are divided:

rf....diode forward resistance.

rb....diode backward resistance

$$\left(\frac{X_0}{A} - X_1\right)\frac{1}{R_1} + \left(\frac{X_0}{A} - X_0\right)\frac{1}{R_0} + \left(\frac{X_0}{A} - E_2 - X_0\right)\frac{1}{rd_2} + \left(\frac{X_0}{A} + E_1 - X_0\right)\frac{1}{rd_1} = 0$$

$$X_0 = \frac{\frac{1}{R_1}X_1 + \frac{E_2}{rd_2} - \frac{E_1}{rd_1}}{\frac{1}{A}\left(\frac{1}{R_1} + \frac{1}{R_0} + \frac{1}{rd_1} + \frac{1}{rd_2}\right) - \frac{1}{R_0} + \frac{1}{rd_1} + \frac{1}{rd_2}}$$

$$X_0 \approx - \frac{\frac{1}{R_1}X_1 + \frac{E_2}{rd_2} - \frac{E_1}{rd_1}}{\frac{1}{R_1} + \frac{1}{rd_1} + \frac{1}{rd_2}}$$

case 1)

$$-E_2 < X_0 < +E_1$$

$$rd_2 = rd_1 = r_b \approx \infty$$

$$-X_0 = \frac{R_0}{R_1}X_1$$

case 2) $X_0 < -E_2$

$$rd_1 = r_b \approx \infty \qquad rd_2 = r_f \approx 0$$

$$-X_0 = \frac{\frac{rd_2}{R_1}X_1 - \frac{rd_2}{rd_1}E_1 + E_2}{1 + \frac{rd_2}{rd_1} + \frac{rd_2}{R_0}} \cong E_2$$

case 3) $X_0 > +E_1$

$$rd_2 = r_b \approx \infty \qquad rd_1 = r_f \approx 0 \qquad -X_0 \cong -E_1$$

The result of this circuit is the negative of the desired relation shown in Fig. 1-a.

ii. Circuit utilizing power sources and potentiometers (Fig. 1-c).

Considering the upper diode while not conducting, the cathode potential "P" is:

$$P = \left[\frac{E}{(1-a_2)r} + \frac{X_0}{a_2 r}\right]\left[(1-a_2)a_2 r\right]$$

$$= \left[a_2 E + (1-a_2)X_0\right]$$

When P equals to zero, it conducts.

$$a_2 E + (1-a_2)X_0 = 0$$

$$X_0 = \frac{a_2}{1-a_2}E$$

Once the upper diode conducts,

$$X_0 = -\left[\frac{1}{R_1}\ \frac{a_2 r}{1 + \frac{a_2 r}{R_0}}\right]X_1 - \frac{a_2}{1-a_2}E$$

Provided the diode forward resistance is neglegible with respect to other resistive parameters.

If $\quad R_1 \gg a_2 r \ , \quad R_0 \gg a_2 r$

or $\quad \frac{a_2 r}{R_1} \ll 1 \ , \quad \frac{a_2 r}{R_0} \ll 1$

$$X_0 \approx -\frac{a_2}{1-a_2}E$$

And saturation voltages E1, E2 will be:

$$E_1 = \frac{a_2}{1-a_2}E$$

(c) Static characteristics.

Static characteristic curves were taken with the circuit of Fig. 1-b, where: $\quad R_1 = R_0 = 1\ M.$

$$E_1 = E_2 = 45\ v.$$

i. Input vs. output plot.

This plot was obtained by use of an X-Y plotter, with the input sine wave frequency of .01 cps. and the result is shown in Fig. 1-d. The choice of input frequency was purely for convenience for the plotter.

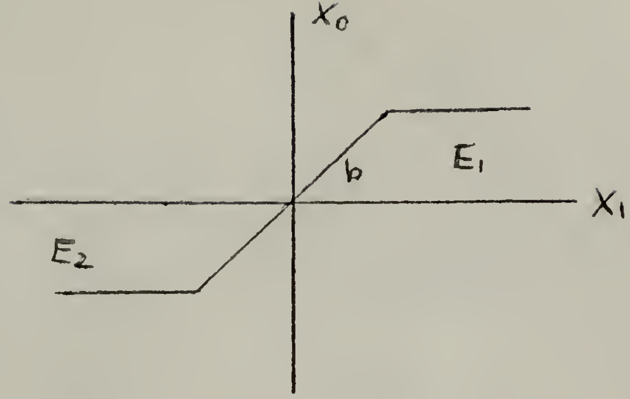


Fig. 1-a   Saturation characteristic

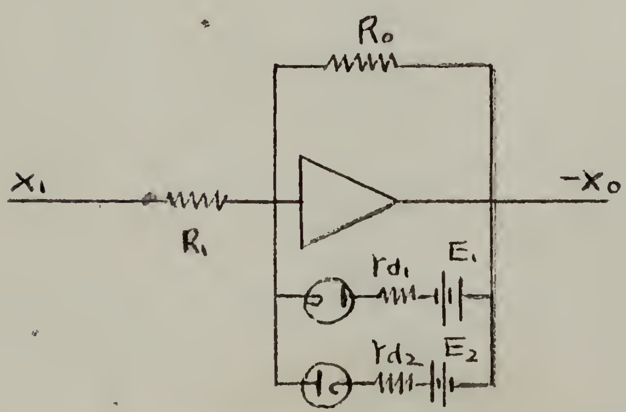


Fig. 1-b Simulation circuit for saturation

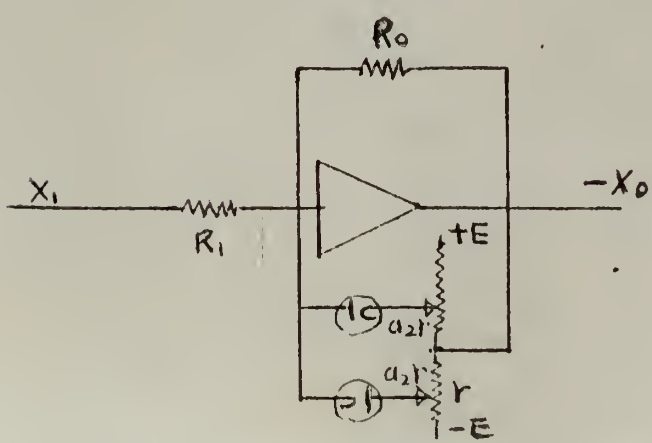
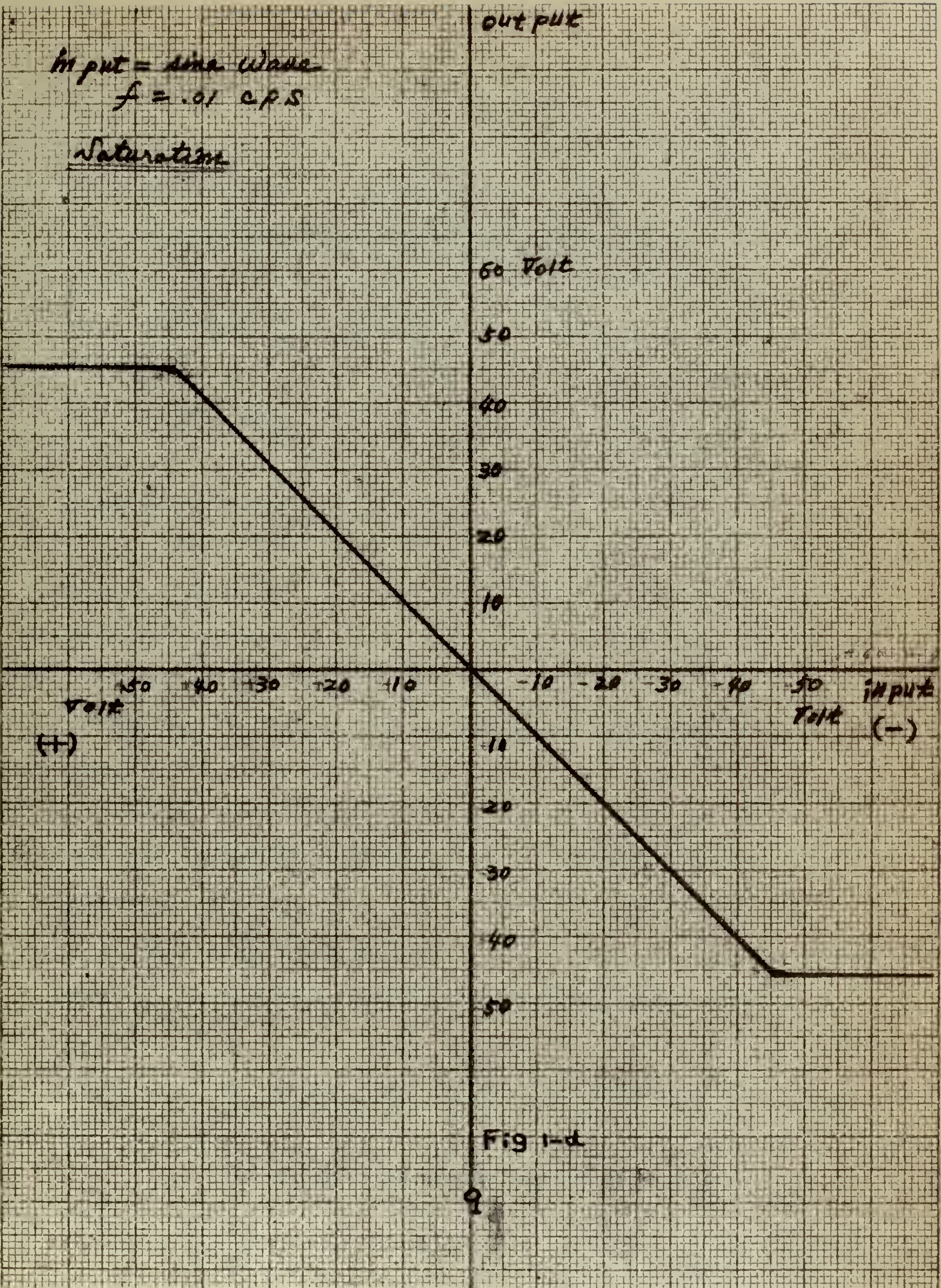


Fig. 1-c Simulation circuit for saturation



output
Input = sine Wave
f = .01 c.p.s
Saturation
60 Volt
50
40
30
20
10
+50 +40 +30 +20 +10
-10 -20 -30 -40 -50
input
Volt
(+)
Volt
(-)
-10
20
30
40
50

Fig 1-d

B. Dead space.

(a) Free play or dead space about the reference point of the load is represented by input vs. output relations as shown in(Fig. 2-a.).

(b) Circuit to represent dead space.

i. Circuit utilizing batteries (Fig. 2-b).

let $E_1 = E_2 = \frac{D}{2}$ $\quad \frac{R_0}{R_1} = b$

case 1) $|E_1|, |E_2| > X_1$

diodes do not conduct.

$$\therefore \quad X_0 = 0$$

case 2) $X_1 < -E_2$

lower diode conducts.

$$X_0 = -\frac{R_0}{R_1}(-X_1 + E_2)$$

provided that the diode and battery resistances are negligible with respect to R1 and R0.

case 3) $X_1 > E_1$

upper diode conducts.

$$X_0 = -\frac{R_0}{R_1}(X_1 - E_1)$$

provided the diode and battery resistances are negligible compared to R1 and R0.

The result is the negative of the desired relation as shown in Fig. 2-a.

ii. Circuit utilizing voltage sources and potentiometers (Fig, 2-c).

Considering upper diode only, if the diode does not conduct, the cathode potential "P" is;

$$P = \frac{(1-a_1)r(a_1 r)}{r}\left[\frac{E}{(1-a_1)r} + \frac{X_1}{a_1 r}\right]$$

$$= a_1 E + (1-a_1)X_1$$

When P equals zero, the diode conducts.

$$a_1 E + (1-a_1)X_1 = 0$$

$$X_1 = -\frac{a_1}{1-a_1}E$$

while the diode is conducting.

$$(X_1' + E)\frac{1}{(1-a_1)r} + (X_1' - X_1)\frac{1}{a_1 r} + (X_1' - \frac{X_0}{A})\frac{1}{R_1} = 0$$

$$X_1 \approx \frac{\frac{X_1}{a_1 r} - \frac{E}{(1-a_1)r}}{\frac{1}{(1-a_1)r} + \frac{1}{a_1 r} + \frac{1}{R_1}}$$

but

$$X_0 = -\frac{R_0}{R_1}X_1'$$

$$= -\frac{R_0}{R_1}\frac{1}{1 + \frac{a_1}{1-a_1} + \frac{a_1 r}{R_1}}\left[X_1 - \frac{a_1}{1-a_1}E\right]$$

if

$$\frac{a_1}{1-a_1} \ll 1^{*} \qquad \frac{a_1 r}{R_1} \ll 1$$

$$X_0 \approx -\frac{R_0}{R_1}\left(X_1 - \frac{a_1}{1-a_1}E\right)$$

and

$$E_1 = \frac{a_1}{1-a_1}E$$

$$E_2 = \frac{a_2}{1-a_2}E$$

*If slope b is defined as;

$$b \triangleq \frac{R_0}{R_1}\frac{1}{1 + \frac{a_1}{1-a_1} + \frac{a_1 r}{R_1}}$$

Then the solution will be theoretically correct. It was assumed that the "a"'s were not corrected for potentiometer loading effects.

(c) Static characteristic curves.

Static characteristic curves were taken with the circuit of Fig. 2-b, where;

$$X_1 = 70 \sin \omega t$$

$$R_1 = R_0 = 1 \text{ M.}$$

$$E_1 = E_2 = 22.5 \text{ V.}$$

i. Input vs. output plot.

This plot was obtained by use of X-Y plotter. Input sine wave frequency of .01 cps. were chosen for pure convenience in the use of the plotter. Result is shown in Fig. 2-d.

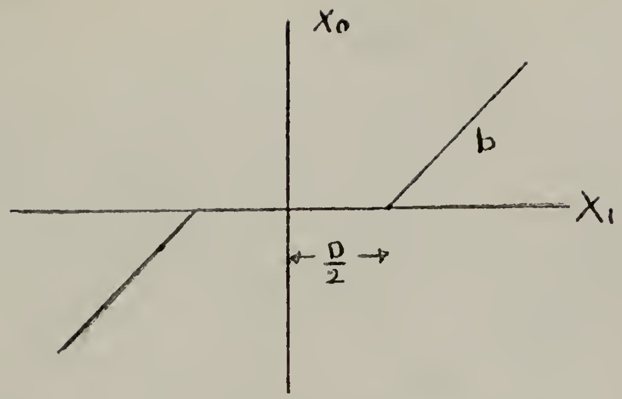


Fig. 2-a Dead space characteristic

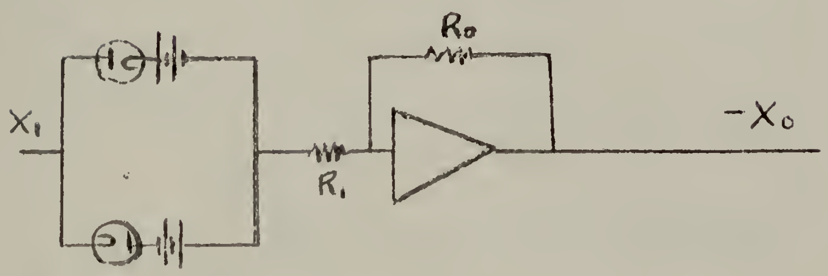


Fig. 2-b Simulation circuit for dead space

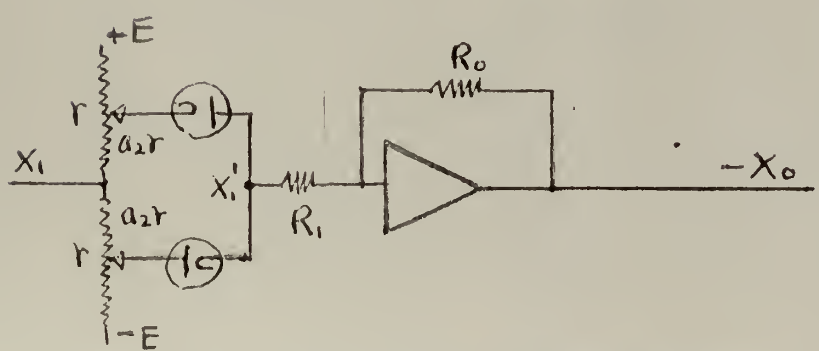


Fig. 2-c Simulation circuit for dead space

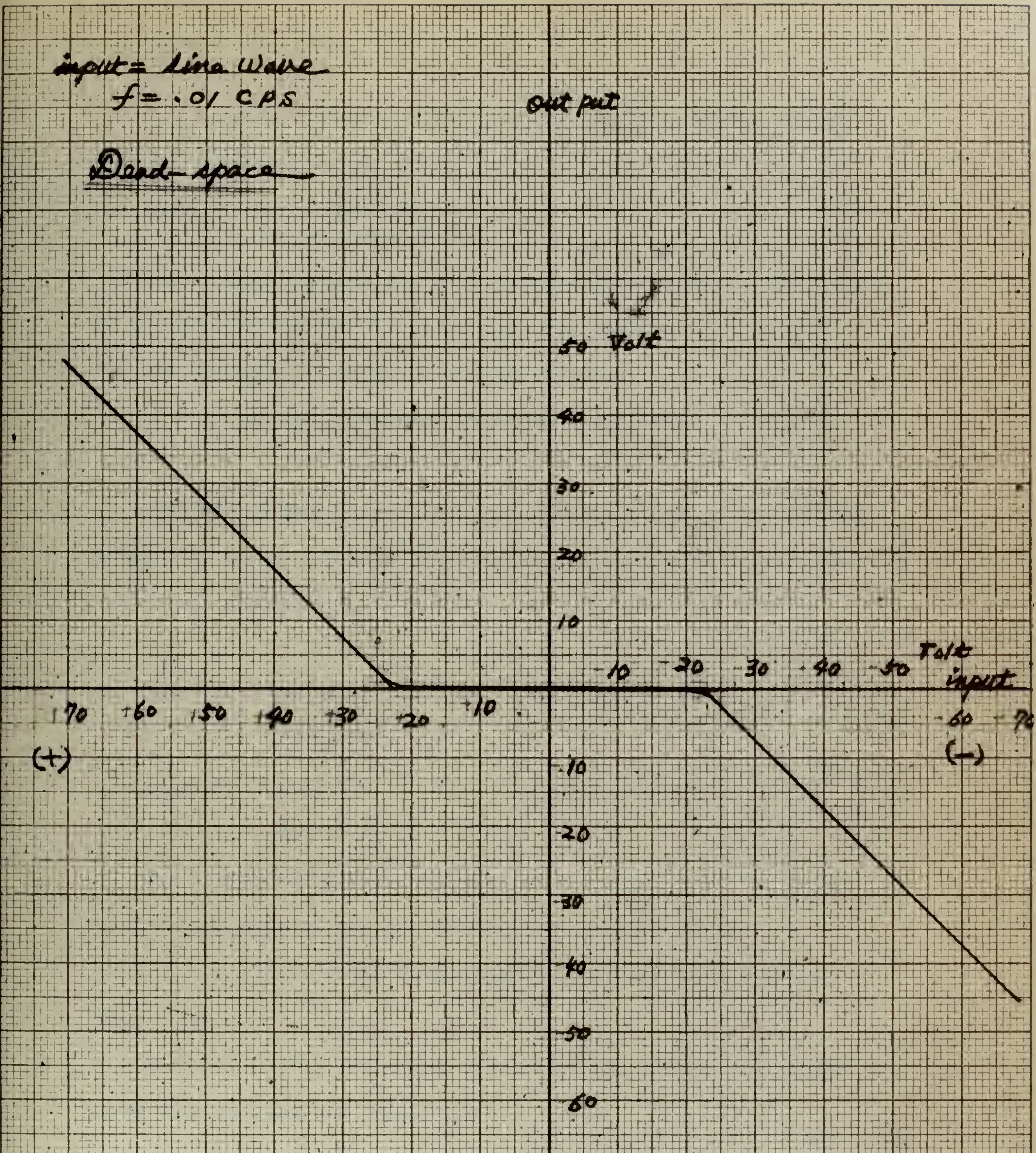

input = sine wave
f = .01 cps
Dead-space
out put
50 Volt
40
30
20
10
10
20
30
40
50
Volt
input
+70
+60
+50
+40
+30
+20
+10
60
70
(+)
(−)
10
20
30
40
50
60

Fig 2-d

C. Coulomb friction.

(a) Coulomb friction at motor or load shaft is represented by the relation shown in Fig. 3-a.

Where; X1 is velocity,
X0 is coulomb force.

The coulomb friction can be referenced to the load or motor shaft as desired, and the simulation is basically similar.

(b) Circuit to represent coulomb friction.

i. Circuit utilizing batteries (Fig. 3-b).

Analysis of this circuit follows closely to that of the one shown in Fig. 1-c except the term $\frac{R_0}{R_1}$ goes to infinity in this case, and the outputs are shown below.

case 1) $X_1 > 0$

$X_0 = -E$

case 2) $X_1 < 0$

$X_0 = +E$

Where diode and battery resistances are neglected. The result is then, the negative of the desired relation of Fig. 3-a.

ii. Circuit utilizing power sources and potentiometers (Fig. 3-c).

Analysis of this circuit is similar to the circuit for saturation except the loop gain is infinity.

case 1) $X_1 < 0$

$$X_0 \cong \frac{a_2}{1-a_2} E_2 \quad \text{if} \quad \frac{a_2 r}{R_1} \ll 1$$

case 2) $x_1 > 0$

$$X_0 \simeq -\frac{a_1}{1-a_1} E_1 \quad \text{if} \quad \frac{a_1 r}{R_1} \ll 1$$

(c) Static characteristics.

Static characteristic curves were taken with circuit Fig. 3-b,

where:

$$X_1 = 70 \sin \omega t$$
$$R_1 = 1.0 \text{ M.}$$
$$E_1 = E_2 = 45 \text{ v.}$$

i. Input vs. output plot.

This plot was obtained by the use of X-y plotter, with input sine wave frequency of .01 cps, and the result is shown on Fig. 3-d.

(d) Discussion.

This method of simulation requires velocity to be available. But this is the usual condition in simulation. Due to the open feed back loop at the instant when X1 equals to zero, the gain is infinite. The noise in amplifier will cause diodes to conduct one way or other, making output never to be zero. But this condition will not affect the actual simulation of coulomb friction because the physical system will behave in like manner and the coulomb torque applied in simulated circuit while velocity is zero will not affect the output.

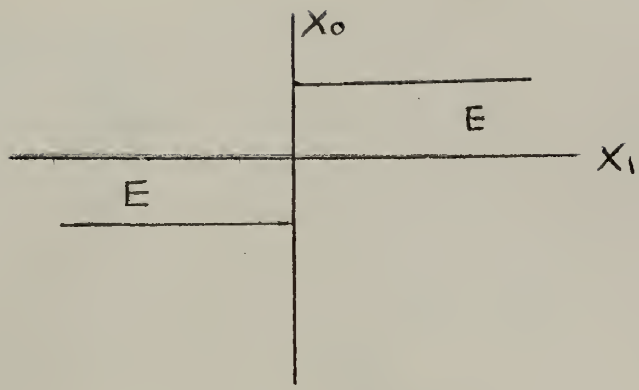


Fig. 3-a   Characteristic of Coulomb friction

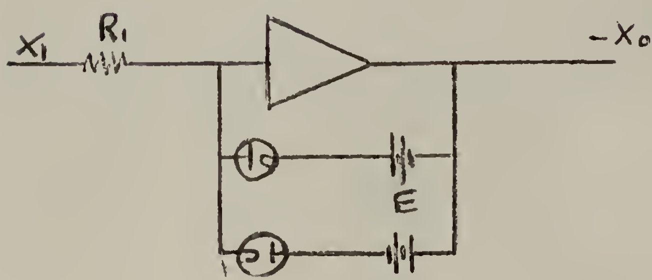


Fig. 3-b   Simulation circuit for Coulomb friction

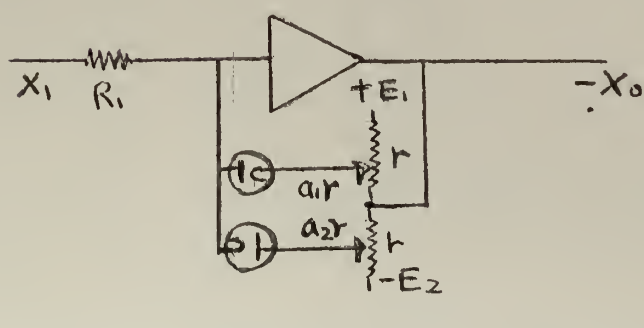


Fig. 3-c   Simulation circuit for Coulomb friction

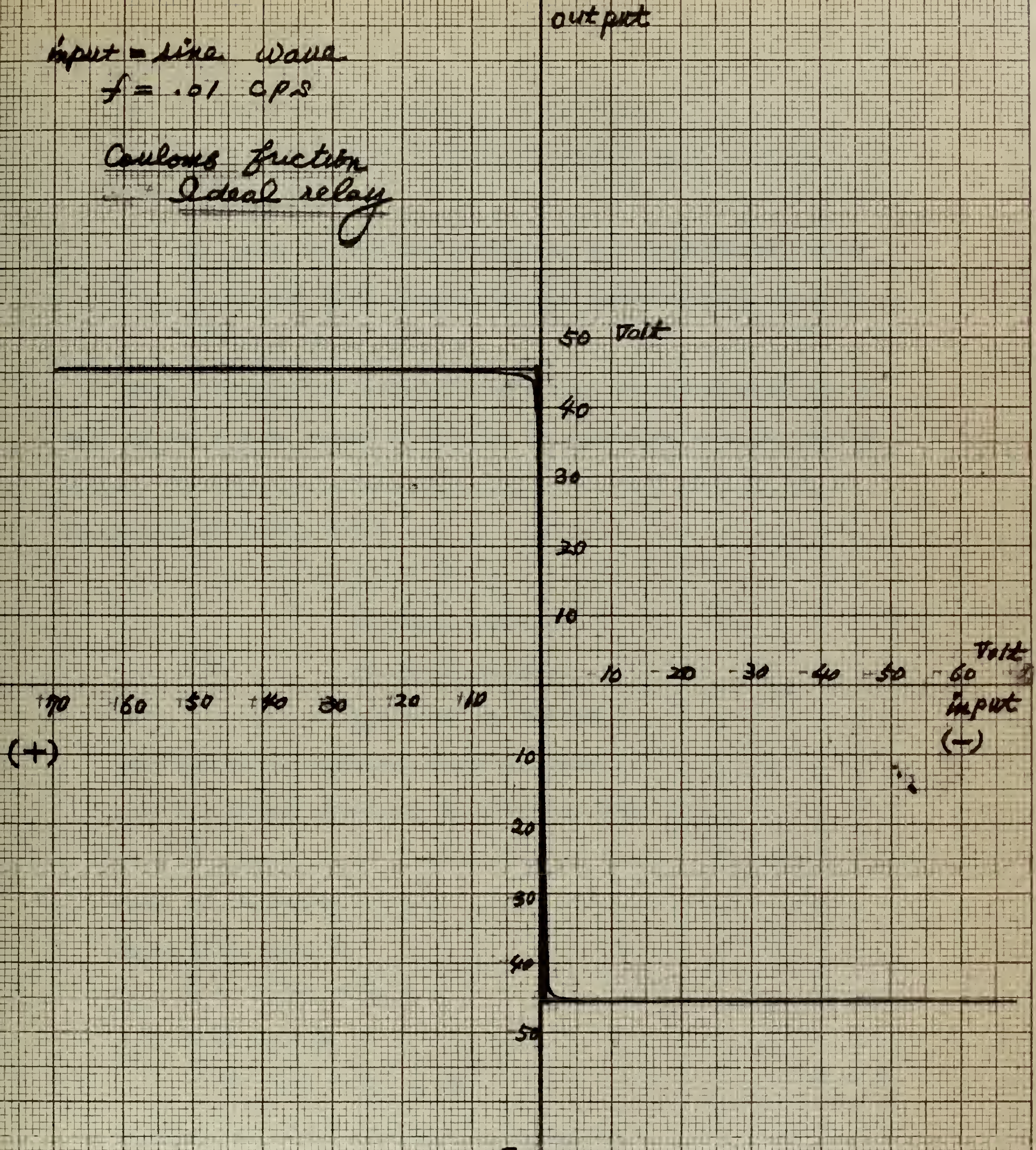

output
input = sine wave
f = .01 cps
Coulomb friction
Ideal relay
50 Volt
40
30
20
10
Volt
10
-20
-30
-40
-50
-60
input
(−)
+70
+60
+50
+40
+30
+20
+10
(+)
10
20
30
40
50

Fig 3-d

D. Simple backlash.

(a) A simple backlash assumes two conditions.

i. Output shaft does not drift when the gear is not in contact.

ii. When the backlash is taken up and recombination occurs, output shaft velocity becomes instantaneously equal to input shaft velocity.

These conditions automatically imply that the output shaft has no mass and has a great amount of friction in comparison with the inertia torque. The relation between input and output shaft position can be represented as in(Fig. 4-a.)

X1..Input position

X0..Output position

Δ ..amount of backlash

(b) Circuit to represent simple backlash.

i. Circuit utilizing dry batteries (Fig. 4-b).

case 1) $Y_1 > +E$

upper diode conducts

$$Y_1 = -(bx_1 + Y_4) \qquad 4.1-1$$

$$Y_2 \cong Y_1 - E$$

$$= -bx_1 - Y_4 + E \qquad 4.1-2$$

Provided the diode resistance is negligible.

$$Y_3 = -\frac{1}{RCS} Y_2 = -\frac{1}{RCS} [-bx_1 - Y_4 + E] \qquad 4.1-3$$

$$Y_4 = -Y_3 = \frac{1}{RCS}\left[-bx_1 - Y_4 + E\right]$$

$$\therefore \quad Y_4\left(1 + \frac{1}{RCS}\right) = \frac{1}{RCS}\left(-bx_1 + E\right)$$

$$Y_4 = \frac{-bx_1 + E}{1 + RCS} \qquad 4.1\text{-}4$$

Substituting eq. 4.1-4 into eq. 4.1-2

$$Y_2 = -bx_1 - Y_4 + E$$

$$= \frac{-bRCSx_1 + RCSE}{1 + RCS} \qquad 4.1\text{-}5$$

$$X_0 = -(Y_2 + Y_4)$$

$$= -\frac{(-bx_1 + E)(1 + RCS)}{1 + RCS}$$

$$= bx_1 - E \qquad 4.1\text{-}6$$

case 2) $Y_1 < -E_1$

Lower diode conducts. The analysis is similar to the case 1, and it eventually yields;

$$X_0 = -bx_1 + E \qquad 4.1\text{-}7$$

case 3) Initially $Y_4 = 0$

$$-E < Y_1 = -bx_1 < +E$$

diodes do not conduct, and $Y_2 = 0$

$$\therefore \quad X_0 = 0$$

After diode has conducted;

$$Y_4 \neq 0$$

and diode cuts off when;

$$-E < -(bx_1 + Y_4) < +E$$

$$\left\{ \begin{array}{l} -E = -(bx_1 + Y_4) \\ +E = -(bx_1 + Y_4) \end{array} \right\} \qquad 4.1\text{-}8$$

From eq. 4.1-4

$$Y_4 = \frac{-bx_1 + E}{1 + RCS}$$

or

$$Y_4 + RCSY_4 = -bx_1 + E$$

$$\therefore \; bx_1 + Y_4 = E - RCSY_4 \qquad 4.1\text{-}9$$

Substituting eq. 4.1-9 into eq. 4.1-8

$$-E = RCSY_4 - E$$

$$RCSY_4 = 0$$

$$\text{or,} \quad \frac{d}{dt}(Y_4) = 0$$

$\therefore$ Upper diode cuts off when Y4 reaches maximum.

$$+E = -RCSY_4 + E$$

$$RCSY_4 = 0$$

$$\frac{d}{dt}(Y_4) = 0$$

$\therefore$ Lower diode cuts off when Y4 reaches maximum in the negative region.

And if $RC \ll 1$

$$\frac{d}{dt}x_1 = 0 \quad \text{when} \quad \frac{d}{dt}Y_4 = 0$$

So, the above condition can be modified to say that when X1 reaches maximum the diodes cut off provided RC is very small in comparison with unity. When diode cuts off $Y_2 = 0$,

$Y_3 = -Y_4 =$ constant at it's maximum value due to charges in the feed back capacitor C, and XO remains constant at the maximum value.

slope b = gain constant of the first amplifier.

backlash $\Delta = \frac{2E}{b}$

ii. Circuit utilizing power sources and potentiometers, instead of the battery (Fig. 4-c).

case 1) $Y_1 > E$

Upper diode conducts

$$Y_1 = -(bx_1 + Y_4) \qquad 4.2\text{-}1$$

$$Y_2 \cong \frac{Y_1 - \frac{a}{(1-a)r}E}{\frac{1}{(1-a)r} + \frac{1}{ar} + \frac{1}{R_1} + \frac{1}{R_2}} \qquad 4.2\text{-}2$$

provided $A \approx \infty$ and diode resistance is negligible.

$$Y_2 \cong \frac{Y_1 - \frac{a}{1-a}E}{1 + \frac{a}{1-a} + \frac{ar}{R_1} + \frac{ar}{R_2}} \qquad 4.2\text{-}3$$

Let

$$\delta \triangleq 1 + \frac{a}{1-a} + \frac{ar}{R_1} + \frac{ar}{R_2}$$

Then

$$Y_2 = \frac{1}{\delta}\left(Y_1 - \frac{a}{1-a}E\right) \qquad 4.2\text{-}4$$

$$Y_4 = \frac{-bx_1 + \frac{a}{1-a}E}{1 + \delta R_1 CS} \qquad 4.2\text{-}5$$

Substituting eq. 4.2-5 in eq. 4.2-4

$$Y_2 = \frac{1}{\delta}\left(Y_1 - \frac{a}{1-a}E\right)$$

$$= \frac{1}{\delta}\left(-bx_1 - Y_4 + \frac{a}{1-a}E\right)$$

$$= \frac{-R_1CS\,bx_1 + R_1CS\,\frac{a}{1-a}E}{1 + \delta R_1CS}$$

$$X_0 = -(Y_2 + Y_4)$$

$$= -\frac{R_1CS\,bx_1 + R_1CS\,\frac{a}{1-a}E - bx_1 + \frac{a}{1-a}E}{1 + \delta R_1CS}$$

$$= \frac{(1 + R_1CS)\left(-bx_1 + \frac{a}{1-a}E\right)}{1 + \delta R_1CS}$$

If $\delta \approx 1$ (see discussion), $X_0 \approx -bX_1 + \frac{a}{1-a}E$

case 2) $Y_1 < -E$

Lower diode conducts, and the analysis is similar to case 1, and it yields:

$$X_0 \approx bx_1 - \frac{a}{1-a}E$$

case 3) Initially $Y_4 = 0$

$$-E < Y_1 = -bx_1 < E$$

diodes do not conduct.

$$X_0 = 0$$

Once a diode has conducted,

$$Y_4 \neq 0$$

When

$$-\frac{a}{1-a}E < -(bx_1 + Y_4) < +\frac{a}{1-a}E \qquad 4.2\text{-}6$$

diode cuts off.

From eq. 4.2-5

$$Y_4 = \frac{-bX_1 + \frac{a}{1-a}E}{1 + \delta R_1 CS}$$

$$Y_4 + \delta R_1 CS = -bX_1 + \frac{a}{1-a}E \qquad 4.2\text{-}7$$

$$Y_4 + bX_1 = \frac{a}{1-a}E - \delta R_1 CS Y_4 \qquad 4.2\text{-}8$$

Comparing eq. 4.2-8 and eq. 4.2-6,

if $\quad \delta R_1 CS Y_4 = 0$

or Y4 reaches maximum value, upper diode cuts off.

if $\quad \delta R_1 C \ll 1$

$$\frac{d}{dt} Y_4 = 0 \quad \text{when} \quad \frac{d}{dt} X_1 = 0$$

If Y4 reaches negative maximum, lower diode cuts off.

$$\frac{d}{dt} Y_4 = 0 \quad \text{when} \quad \frac{d}{dt} X_1 = 0$$

slope b = gain constant of the first amplifier.

backlash $\quad \Delta = \frac{2a}{b(1-a)}E$

provided $\quad \delta \approx 1$

(c) Static characteristics.

Static characteristic curves were taken with circuit shown in Fig. 4-c, where;

$X_1 = 70 \sin \omega t$

$b = 1$

$\bar{\Delta} = 8$ v.

$C \cong 1 \mu f$

$R_1 \approx .05$ M.

$E = 45\ V$

$r = 100\ K$

$a = .151$ ---------- (this value could not be made lower due to non availability of higher voltage battery.)

i. Input vs. output plot.

This curve was obtained through the use of X-Y plotter. Input sine wave frequency of .01 cps. was chosen for the convenience of plotting, and the result is shown on Fig. 4-d.

(d) Discussion.

In any simulation of a servo system, the output velocity must be obtainable. In this backlash case, one has to have a differentiator just after the output Xo in order to obtain the output velocity. To avoid the erroneous result of the differentiator, one usually has to have some small resistance connected in series with the input capacitor if the input is of the step nature. Once a resistance is connected in the input side of the amplifier, the differentiated quantity will have error. If such is the case and the RCS in the denominator could be tolerated, the out put velocity is directly available in this simulation circuit. Y2 in Fig. 4-b and 4-c is the one, ie:

From eq. 4.1-5

$$Y_2 = \frac{-bRCSX_1}{1 + RCS} \qquad \because RCSE = 0$$

if $RC \ll 1$

$$Y_2 \simeq -bRCX_1$$

(while diode is conducting)

$$Y_2 = 0 \qquad \text{(while diodes are cut off)}$$

So, the voltage $Y2$ represents approximately the output velocity. The quanity δ.

$$\delta \triangleq 1 + \frac{a}{1-a} + \frac{ar}{R_1} + \frac{ar}{R_2}$$

$$\delta \approx 1 \quad \text{if} \quad \frac{a}{1-a} \ll 1 \quad \frac{ar}{R_1} \ll 1 \quad \frac{ar}{R_2} \ll 1$$

(It has been assumed that the potentiometer loading effect is not corrected.)

The above condition can be realized with the use of higher voltage sources and lower resistance potentiometers with respect to the value of R_1.

Also, if a corrected potentiometer setting is chosen, and if

$$\frac{a}{1-a} \ll 1$$

$$\delta \approx 1$$

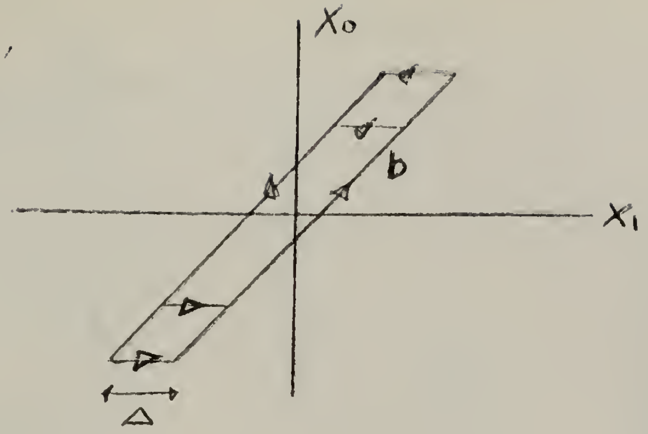


Fig. 4-a   Characteristic of simple backlash

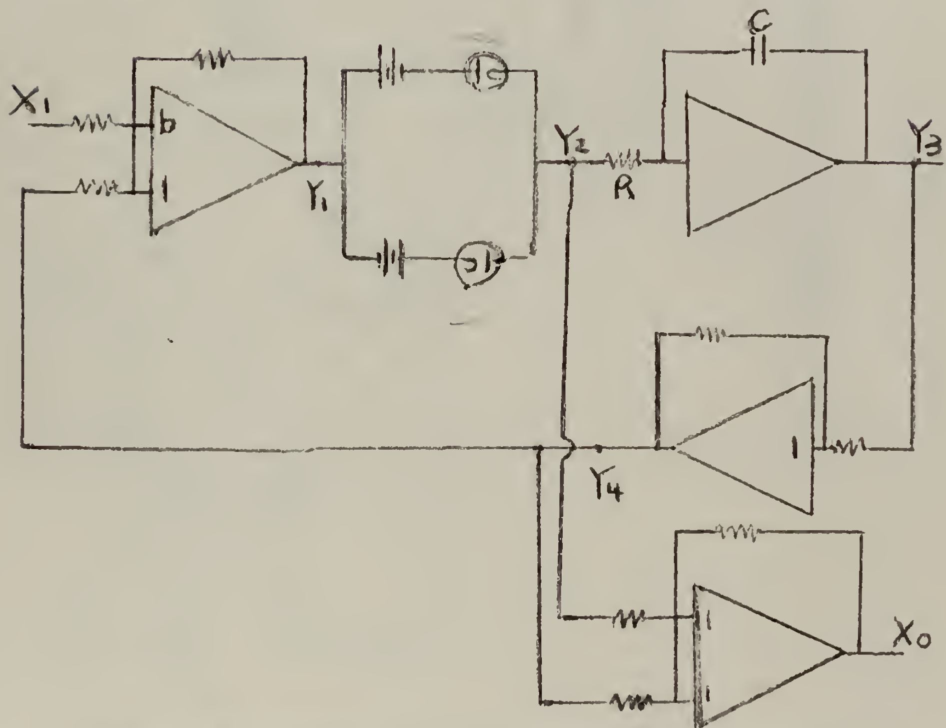


Fig. 4-b   Simulation circuit for simple backlash

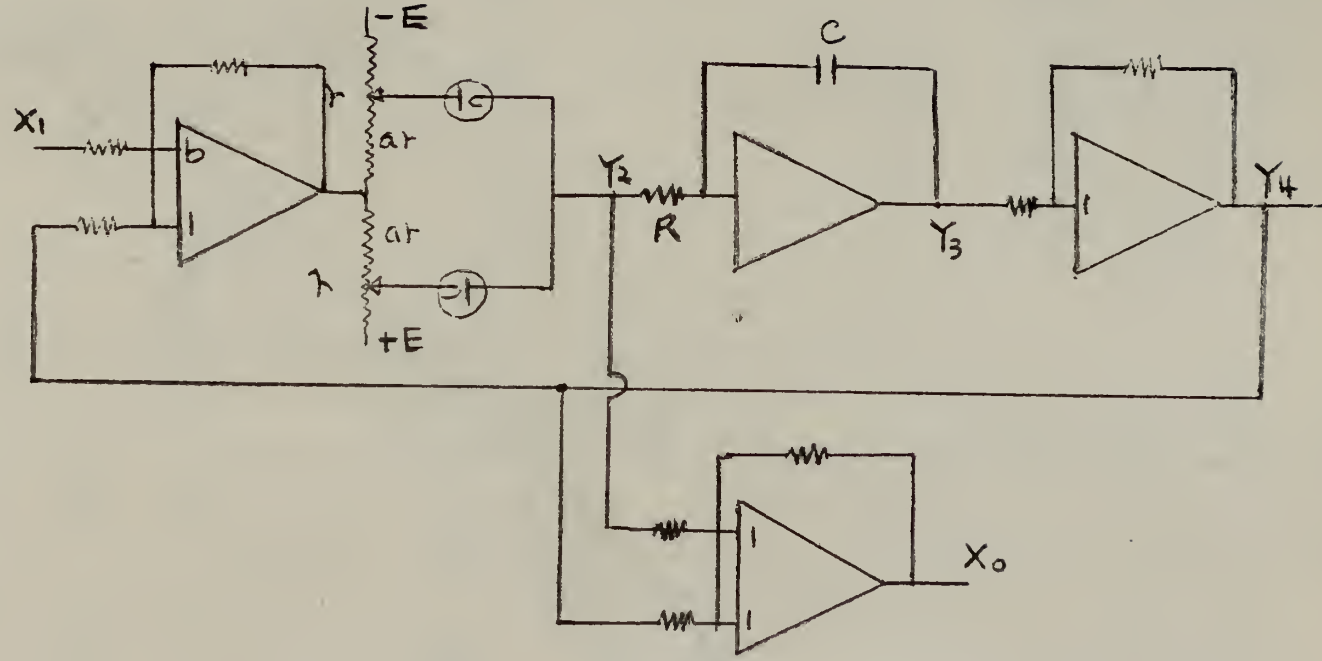
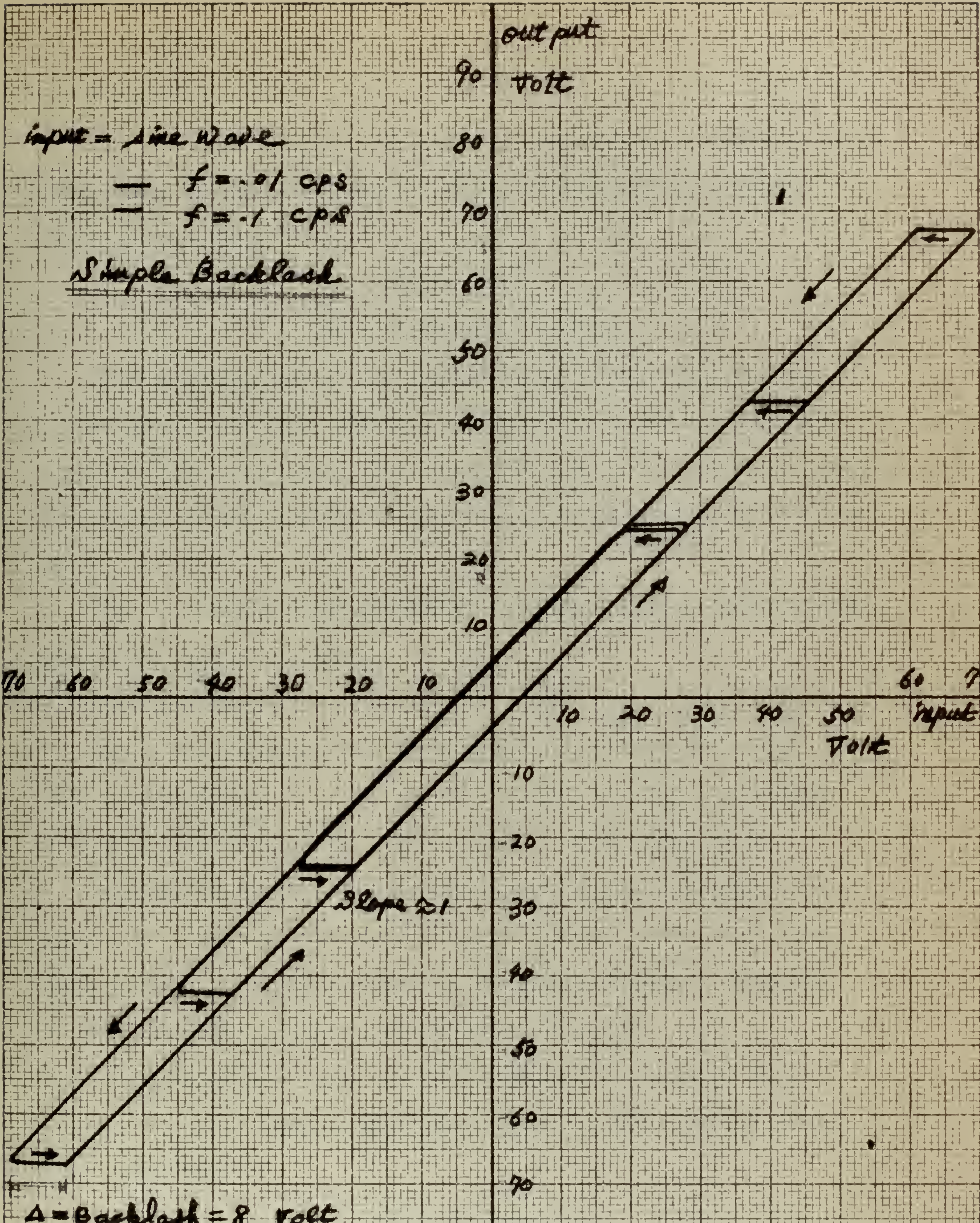


Fig. 4-c Simulation circuit for simple backlash



output
Volt
input = sine wave
f = .01 cps
f = .1 cps
Simple Backlash
Slope ≅ 1
input
Volt
Δ = Backlash = 8 volt

Fig. 4-d

E. Ideal relay.

(a) An ideal relay has no dead space and acts on an infinitesimal amount of the error signal. It can be represented by the relation shown in Fig. 5-9.

X1....Error signal.

X0....Relay output voltage

This curve has exactly the same relation between input and output as coulomb friction. The only difference is in the quantity that input and output represent. For this reason the circuit used for simulation of the coulomb friction is also used as a simulator of an ideal relay.

(b) For circuit; analysis of it, and static curves, refer to paragraph C of coulomb friction.

(c) Discussion.

Due to the infinite gain in the open loop, the output voltage of this circuit will be E1 or E2 even if the input is zero, which seems to be contradictional to the characteristics of the ideal relay.

In order to remedy this situation, reducing the loop gain has been tried. But then also, this will not be a true simulation of an ideal relay because of the finite slope in the output voltage to reach the desired level of E1 or E2 as illustrated in Fig. 5-6.

If E1 and E2 are small, and if the gain is sufficiently large so that the finite slope is tolerable, there are three ways to obtain such a high gain.

i. Decrease R1. (Refer to Fig. 5-c.)

This method seemed to be the best way to get high gain without introducing more noises among three methods. But R1 can not be decreased too low if voltage sources and potentiometers are used in place of battery as shown in Fig. 5-d.

if $\frac{R_0}{R_1} \gg 1$ say about 500;

$$X_0 = \frac{ar}{R_1} x_1 \pm \frac{a}{1-a} E$$

and $\frac{ar}{R_1}$ may become not negligible.

ii. Increase R0.

Since the maximum value of resistance available is limited, writer had to connect many resistors in series in the feed back path. This situation was rather an awkward one. It also tended to increase the noise effects.

iii. Fractional output voltage feed back method. (Refer to Fig. 5-e).

$$X_0 \approx -\frac{R_0}{R_1}\frac{1}{a}X_1$$

With the potentiometer coefficient "a" very small, the gain can be boosted to a high value. But, for some reason, this method had more noise effect than either of the two methods discussed previously.

Taking these factors in consideration, plus the fact that the physical system will also have noise such that the output will never become zero, writer has chosen the open loop circuit for simulation of an ideal relay.

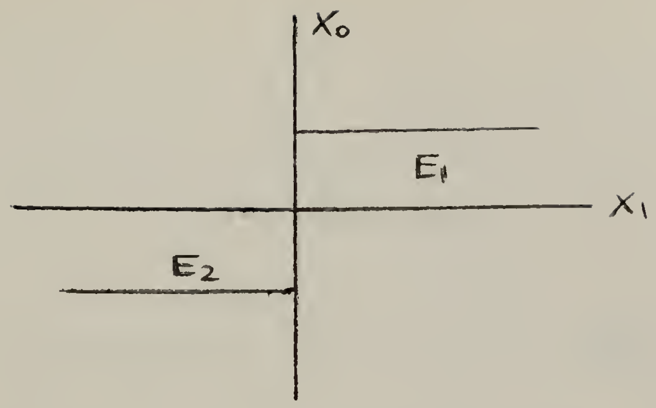


Fig. 5-a Characteristic of ideal relay

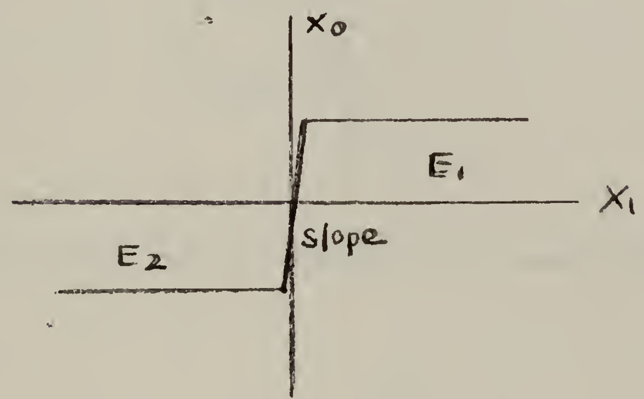
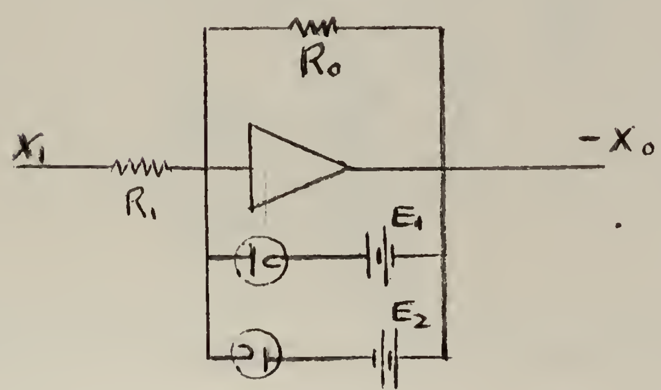
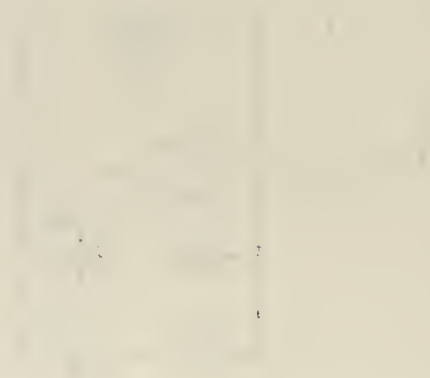


Fig. 5-b Simulation circuit for ideal relay



Fif. 5-c Simulation circuit for ideal relay



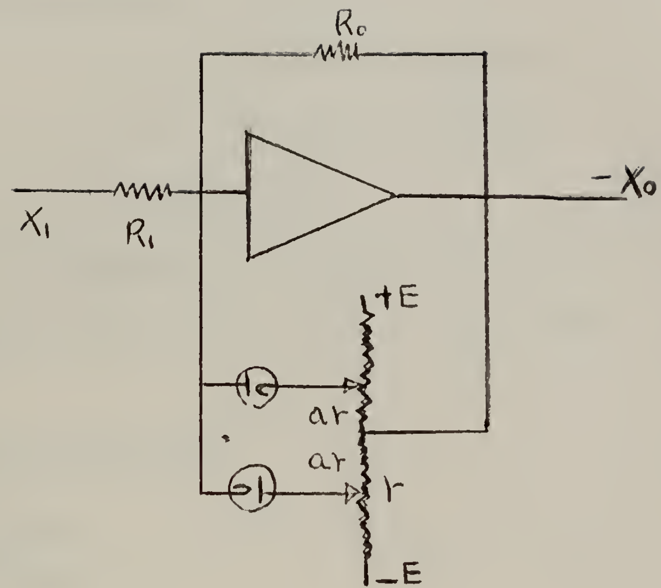


Fig. 5-d Simulation circuit for ideal relay

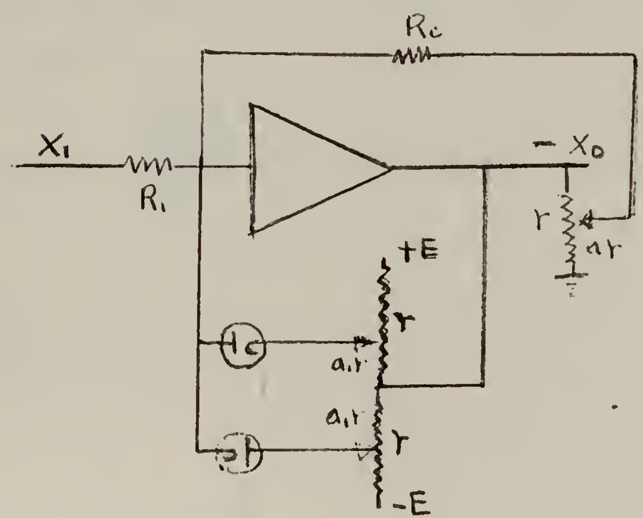


Fig. 5-e Simulation circuit for ideal relay

F. Relay with dead zone.

(a) Relay with dead zone but no hysteresis effect, has the input vs. output relation as shown in Fig. 6-a.

X1....Relay input voltage.

X0....Relay output voltage.

D....Dead zone.

(b) Circuit to represent the relation of Fig. 6-a.

i. Circuit using dry batteries. (Fig. 6-b).

case 1) $-E < X_1 < +E$

$$Y_1 = 0$$
$$X_0 = 0$$

case 2) $\frac{D}{2} = +E < X_1$

$$Y_1 = X_1 - E$$

$$X_0 = -\frac{R_0}{R_1}(X_1 - E) \quad \text{until} \quad -\frac{R_0}{R_1}(X_1 - E) \Rightarrow E_1$$

if X0 becomes E1;

$$x_0 = -\frac{\frac{R_0 r_f}{R_0 + r_f}}{R_1}(X_1 - E) - E_1$$

if $\frac{R_0 r_f}{R_0 + r_f} \approx 0$

$$X_0 \approx -E_1$$

case 3) $X_1 < -E = -\frac{D}{2}$

$$Y_1 = -X_1 + E$$

$$X_0 = \frac{R_0}{R_1}(X_1 - E) \quad \text{until} \quad \frac{R_0}{R_1}(X_1 - E) \Rightarrow E_2$$

if $$X_0 = \frac{R_0}{R_1}(X_1 - E) > E_2$$

$$X_0 = \frac{\frac{R_0 r_f}{R_0 + r_f}}{R_1}(X_1 - E) + E_2 \approx E_2$$

provided $\frac{r_f}{R_0} \approx 0$

Result of this circuit will be the negative of the desired characteristics if $\frac{R_0}{R_1} >> 1$

ii. Circuit using power sources and potentiometers. (Fig. 6-c).

case 1) $\frac{a_3}{1-a_3} E_3 < X_1 < -E_3 \frac{a_3}{1-a_3} = -\frac{D}{2}$

$$Y_1 = \frac{1}{\frac{1}{(1-a_3)r_3} + \frac{1}{a_3 r_3} + \frac{1}{R_1}} \left[\frac{X_1}{a_3 r_3} - \frac{1}{(1-a_3)r_3} E_3 \right]$$

$$= \frac{1}{1 + \frac{a_3}{1-a_3} + \frac{a_3 r_3}{R_1}} \left[X_1 - \frac{a_3}{1-a_3} E_3 \right]$$

Let

$$\delta_3 \triangleq 1 + \frac{a_3}{1-a_3} + \frac{a_3 r_3}{R_1}$$

Then

$$Y_1 = \frac{1}{\delta_3}\left(X_1 - \frac{a_3}{1-a_3} E_3 \right) \approx X_1 - \frac{a_3}{1-a_3} E_3$$

$$\text{if } \delta_3 \approx 1$$

$$X_0 \approx -\frac{R_0}{R_1}\left(X_1 - \frac{a_3}{1-a_3} E_3\right) \text{until} \quad X_0 \Rightarrow \frac{a_4}{1-a_4} E_4$$

If $|X_0|$ reaches $\frac{a_4}{1-a_4} E_4$

Then $$X_0 = \pm \frac{\dfrac{a_4 r_4 R_0}{R_0 + a_4 r_4}}{R_1}\left(X_1 - \frac{a_3}{1-a_3}E_3\right) \pm \frac{a_4}{1-a_4}E_4$$

$$\approx \frac{a_4}{1-a_4}E_4$$

provided

$$\frac{a_4 r_4}{R_0} \ll 1, \qquad \frac{a_4 r_4}{R_1} \ll 1$$

case 2) $$-E_3\frac{a_3}{1-a_3} < Y_1 < E_3\frac{a_3}{1-a_3}$$

$$Y_1 = 0 \qquad X_0 = 0$$

The result of this circuit will approximate the relations shown in Fig. 6-a in the negative sense. (δ_3 does not affect the characteristics of this circuit if the condition $\frac{R_0}{R_1} \gg 1$ is met.)

(c) Static characteristics.

Static characteristic curves were taken with the circuit of Fig. 6-b, where:

$X_1 = 70 \sin \omega t$

$\frac{D}{2} = 22.5$ V.

$E_1 = E_2 = 22.5$ V

gain ≈ 400

$R_1 = .1$ M

$R_0 = 20$ M.

feed back pot. setting a = .5

i. Input vs. output curve.

This curve was obtained by using X-Y plotter with input sine wave frequency of .01 cps. as shown on Fig. 6-d.

From this plot, it can be seen that the diode cut off and conduction voltage has a little difference. If a person wishes to correct this effect, it can be done by using feed back as is shown in para. G of relay with hysseresis.

(d) Discussion.

Aside from errors arising from noncritical components of the

circuit, the most important factor in this circuit for successful operation with reasonable accuracy is to have the loop gain as great as can be obtained. But the boosting of this gain is limited, because of the dead zone associated with relay. If the gain is too high, simulation shows unstability in the dead zone due to noises amplified. If the gain is too low, it would not simulate the desired characteristics of this relay because of the finite time required for output to reach the relay output voltage. So, there is a compromise between these two factors.

The degree of compromise will depend on the computer and the problem itself. The gain was chosen as 400 in this case from the observance of the performance of this circuit on the brush recorder. The output voltage jumped almost vertically to relay output voltage from zero without showing noises in the dead zone, with this gain on the brush recorder. The input voltage was chosen a low time rate of change voltage wave for this observance, it was .01 cps. sine wave of magnitude 25 volts.

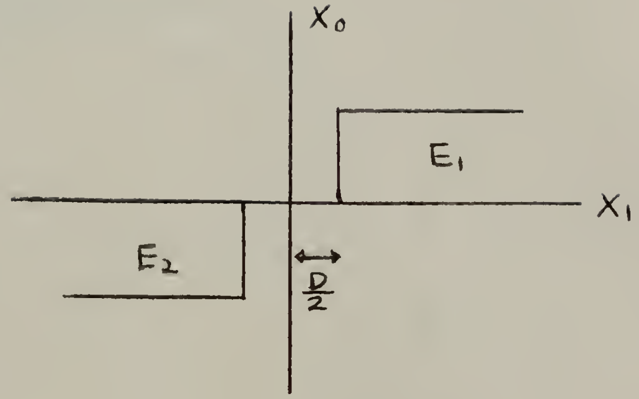


Fig. 6-a   Characteristic of relay with dead zone

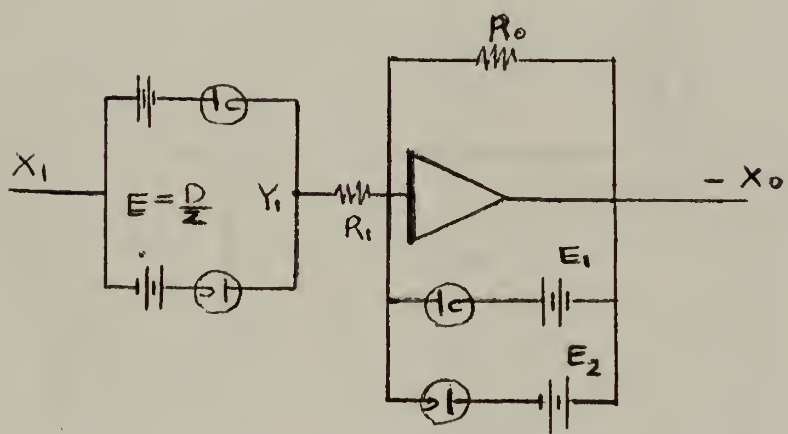


Fig. 6-b Simulation circuit for relay with dead zone

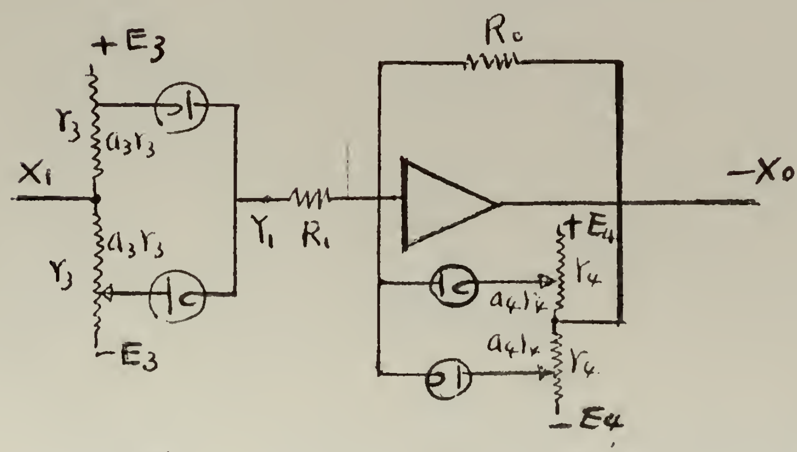


Fig. 6-c   Simulation circuit for relay with dead zone

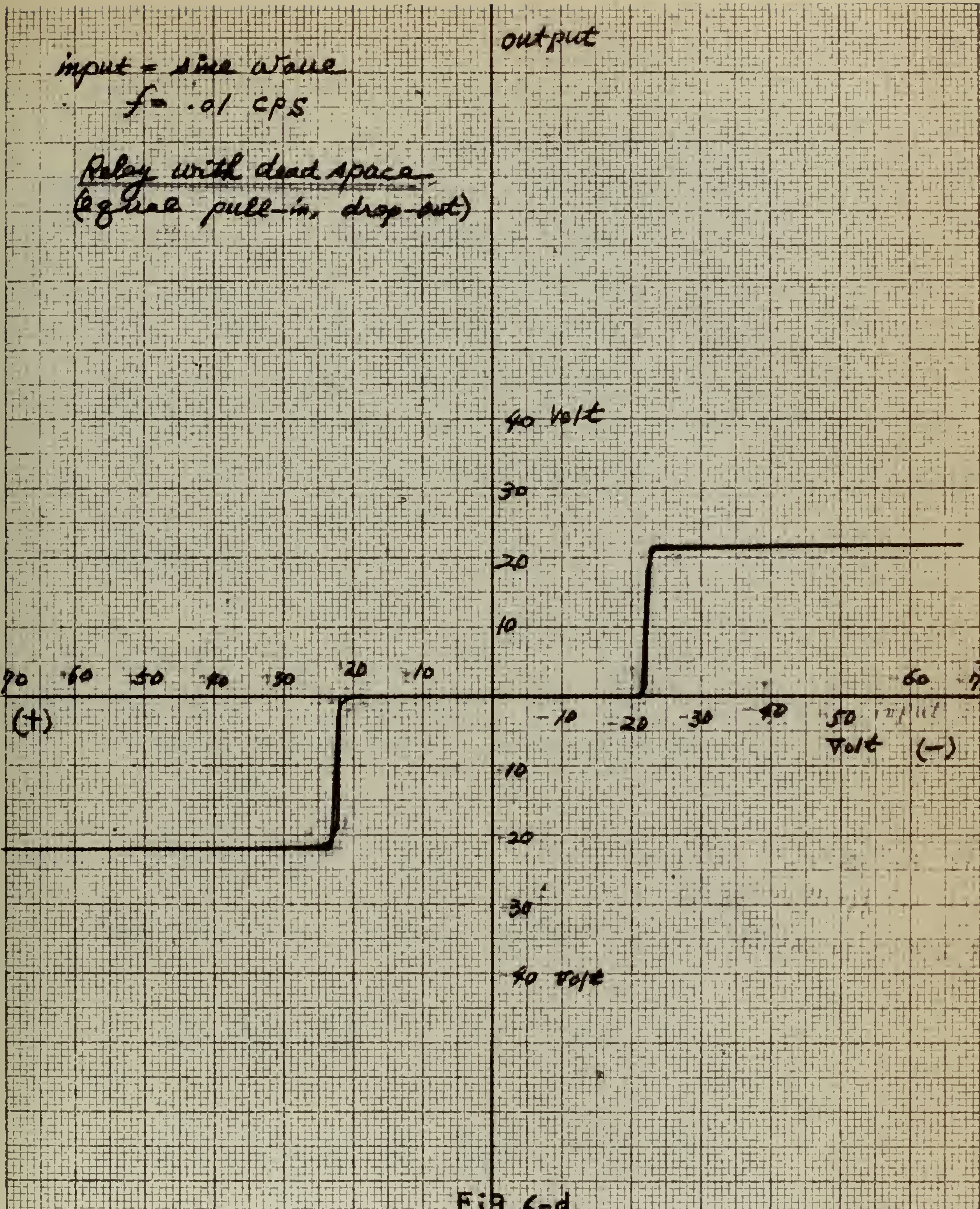

input = sine wave
f = .01 cps
Relay with dead space
(equal pull-in, drop-out)
output
40 Volt
30
20
10
70
60
50
40
30
20
10
(+)
-10
-20
-30
40
50
60
70
Volt
(−)
10
20
30
40 Volt

Fig. 6-d

G. Relay with dead zone and hysteresis.

(a) This relay will have unequal "pull in" and "drop out" voltages, and it has the input vs. output relation shown on Fig. 7-a.

X1....Relay input voltage.

X0....Relay output voltage.

$\frac{p}{2}$...."pull in" voltage.

$\frac{d}{2}$...."drop out" voltage.

(b) Circuit to represent relay with dead zone and hysteresis.

i. Circuit utilizing batteries (Fig. 7-b).

case 1) $+E > X_1 > -E$

Starting from zero,

$$Y_1 = -X_1 \quad Y_2 = 0$$

$$X_0 = 0$$

case 2)

(1) When X1 increases from zero in the positive direction.

$$X_1 > +E$$

$$Y_1 = -X_1$$

$$Y_2 = -X_1 + E = -X_1 + \frac{p}{2}$$

where voltage drop in the diode is neglected.

$$X_0 = \frac{R_0}{R_1}(X_1 - \frac{p}{2})$$ until Xo reaches E1.

Then

$$X_0 \neq 0$$

$$Y_1 = -(X_1 + bX_0) \quad X_0 = \frac{R_0}{R_1}(X_1 + bX_0 - \frac{p}{2})$$

$$X_0 = \frac{1}{1 - \frac{R_0}{R_1}b} \frac{R_0}{R_1}(X_1 - \frac{p}{2}) \qquad 7.1-1$$

For the desired condition that the Xo jumps instantaneously to E1 or E2 from zero, gain $\frac{R_0}{R_1} \gg 1$ as was discussed in previous paragraph. If the amplifier of the computer is a perfect one with no inter capacitance and a perfect 180 degree phase shift, the output Xo will behave as indicated by eq. 7.1-1 during the fraction of a second when Xo is in the jumping stage to E1 or E2.

But, due to the input and output diode limiter, current can not reverse in that same diode, and in the limit, eq. 7.1-1 will behave like eq. 7.1-2.

$$X_0 = \frac{R_0}{R_1}\left(X_1 - \frac{P}{2}\right) \qquad 7.1-2$$

(until Xo reaches E1 or E2)

But, an amplifier can not usually be perfect, then it is most probable that oscillation may occur during this stage, making it unstable. However unstable it may be, oscillation can not occur in this circuit because of two sets of limiting diodes, and this condition will tend to make a steeper slope when Xo jumps to relay output voltage from zero. When Xo reaches E1.

$$X_0 = \frac{\frac{r_f R_0}{R_0 + r_f}}{R_1}\left(X_1 - \frac{P}{2}\right) + E_1$$

where r_f = diode forward resistance.

since $\frac{r_f}{R_0} \ll 1$, $\frac{r_f}{R_1} \ll 1$

$X_0 \approx +E$ at $X_1 = \frac{P}{2} = E$ while X1 is increasing.

(2) After X1 has reached the maximum and decreases in the positive region.

$$X_0 = \frac{\frac{r_f R_0}{R_0 + r_f}}{R_1}\left[X_1 + bE_1 - \frac{P}{2}\right] + E_1 \approx E_1$$

$$Y_2 = X_1 + bE_1$$

$Y_2 < \frac{\rho}{2}$, input diode will cut off,

or, $X_1 + bE_1 < \frac{\rho}{2} = E \qquad X_1 < \frac{\rho}{2} - bE_1$

If this cut off voltage is to be $\frac{d}{2}$,

$$\frac{\rho}{2} - bE_1 = \frac{d}{2}$$

$$b = \frac{\frac{\rho}{2} - \frac{d}{2}}{E_1}$$

$X_0 = +E_1$ for $X_1 > \frac{\rho}{2}$ when X1 increases.

$X_0 = +E_1$ until $X_1 = \frac{d}{2}$ when X1 decreases.

case 3) $X_1 < -E$

Analysis is the same as case 2 except that the other diode conduct and signs reversed.

$X_0 = -E_2$ for when increases.

$X_0 = -E_2$ until when decreases.

ii. Circuit utilizing power sources and potentiometers. (Fig. 7-c).

Starting from X1 = 0 and increases in the positive region.

$$X_1 = 0$$

$$Y_1 = -X_1$$

for $-\frac{a_3}{1-a_3}E_3 < X_1 < +\frac{a_3}{1-a_3}E_3$

$$Y_2 = 0$$

$$X_0 = 0$$

when $X_1 > \frac{a_3}{1-a_3}E_3$, $\quad Y_2 = \frac{1}{1 + \frac{a_3}{1-a_3} + \frac{a_3 r_3}{R_1}}\left[X_1 - \frac{a_3}{1-a_3}E_3\right]$

Let $\dfrac{1}{\delta_3} \triangleq \dfrac{1}{1 + \dfrac{a_3}{1-a_3} + \dfrac{a_3 r_3}{R_1}}$

$$Y_2 = -\frac{1}{\delta_3}\left[X_1 - \frac{a_3}{1-a_3}E_3 \right]$$

$$X_0 = \frac{R_0}{R_1}\left[\frac{1}{\delta_3}\left(X_1 - \frac{a_3}{1-a_3} \right) \right]$$

But, if $\frac{R_0}{R_1} \approx \infty$, Xo will reach instantaneously to $\frac{a_4}{1-a_4}E_4$ and Xo will be limited at that value. The possibility of oscillation due to positive feed back is eliminated by diode limiters as discussed in the previous pages.

When Xo reaches instantaneously to:

$$X_0 = \frac{a_4}{1-a_4}E_4$$

$$X_0 = \frac{\dfrac{R_0 a_4 r_4}{R_0 + a_4 r_4}}{R_1}\left(\frac{1}{\delta_3} + b\,\frac{a_4}{1-a_4}E_4 - \frac{a_3}{1-a_3}E_3 \right] + \frac{a_4}{1-a_4}E_4$$

If $\quad \dfrac{a_4 r_4}{R_0} \ll 1 \quad , \quad \dfrac{a_4 r_4}{R_1} \ll 1 \quad , \quad X_0 \approx \dfrac{a_4}{1-a_4}E_4$

After X1 reaches the maximum and decreases in the positive region, the diode in the input limiter will not cut off at $X_1 = \frac{a_3}{1-a_3}E_3$ where it started to conduct while X1 was increasing, due to the positive feed back; ie., a variable DC bias voltage. And it cuts off when;

$$X_1 + b\,\frac{a_4}{1-a_4}E_4 = \frac{a_3}{1-a_3}E_3$$

or

$$X_1 = \frac{a_3}{1-a_3}E_3 - b\,\frac{a_4}{1-a_4}E_4$$

If $\quad \dfrac{a_3}{1-a_3}E_3 = \dfrac{\rho}{2}$

$$b\,\frac{a_4}{1-a_4}E_4 = \frac{\rho}{2} - \frac{d}{2}$$

This circuit will give similar results as the circuit appears on Fig. 7-b, provided following conditions are met.

$$\frac{a_3 r_3}{R_1} \ll 1 , \qquad \frac{a_3 r_3}{R_0} \ll 1 , \qquad \frac{R_0}{R_1} \gg 1$$

(c) Static characteristics.

Static characteristic curve was taken with circuit appearing on Fig. 7-b with input sine wave of frequency .01 cps., where;

$$E = \frac{P}{2} = 22.5 \text{ V.}$$

$$E_1 = E_2 = 22.5 \text{ V}$$

$$R_1 = .1 \text{ M.}$$

$$R_0 = 20 \text{ M.}$$

b = 3 different values. (setting of b was accomplished by setting potentiometer coefficient of Xo, while the gain of first amplifier was maintained at unity).

Magnitude of input sine wave was chosen two different values to see the behavior of the circuit while Xo jumps from zero to E1 or E2, on the brush recorder.

i. Input vs. output plot.

This curve was plotted by X-Y plotter and is shown on Fig. 7-e, where;

$$X_1 = 75 \sin \omega t$$

$$f = .01 \text{ CPS}$$

Fig. 7-3(I)

$$\frac{P}{2} - \frac{d}{2} = 5 \qquad b = \frac{5}{E_1} = .2222$$

Fig. 7-e(II)

$$\frac{P}{2} - \frac{d}{2} = 8 \qquad b = \frac{8}{E_1} = .355$$

Fig. 7-3(III)

$$\frac{P}{2} - \frac{d}{2} = 0 \qquad b = 0$$

ie. a relay with dead space only.

(d) Discussion

i. Since the method is based on using a variable DC bias voltage, the success of the circuit will largely depend on the gain of the second amplifier $\frac{R_0}{R_1}$.

The greater the gain the better the simulation will be. But this gain is limited. Because, for higher gain, the noise amplification becomes great and it appears in the output signal in the dead zone.

In this case gain $\frac{R_0}{R_1} \approx 200$ was chosen and the result was satisfactory. However, there was a constant error in $\frac{P}{2} - \frac{d}{2}$, as indicated on Fig. 7-e.

To investigate this error, one more plot with $b=0$ (without feed back) was taken as appears on Fig. 7-e(III). As can be seen from that figure, it also has about the same amount of error, and the writer believes that the error is most likely caused from the nonideality of diodes.

ii. The error discussed above can be corrected. Fortunately, the error is always outward from the origin, ie., the diode conducting point is greater than the cut off point. Once this error is known to be present, it can be corrected by modifying the potentiometer coefficient setting "b".

iii. A very low time rate of change voltage sine wave input was tested in this circuit to see the slope of Xo when it jumps to output voltage level, on the brush recorder. The result was a vertical slope even if the input was low time rate changing voltage. It is due to the unstability of this positive feed back network while the transition of Xo from zero to output voltage level.

This steeper slope does not imply that the gain $\frac{R_0}{R_1}$ is increased, and it will not give better results as a higher gain would.

Supposing "b" is zero with the feed back loop left in place, the diode must cut off exactly at the point where that diode has conducted. But, due to the finite gain $\frac{R_0}{R_1}$, there will be a finite time required for Xo to reach E1 or E2. In other words, there is a finite change required in X1 in order to let Xo reach E1 or E2. The steeper slope of Xo does not mean that the gain is actually increased. It only means that it shows a steeper slope inside the zone δ, as is described on Fig. 7-d, and it does not make the zone δ narrower, which is the factor that governs the accuracy of the circuit performance. (Refer to Fig. 7-d).

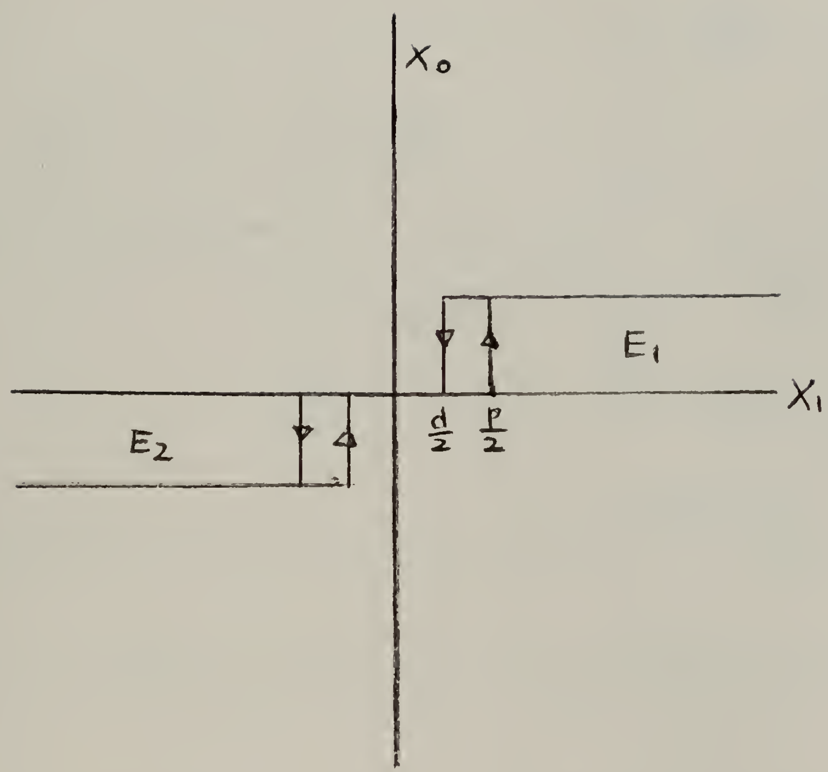


Fig. 7-a Characteristic of relay with dead zone and hysteresis

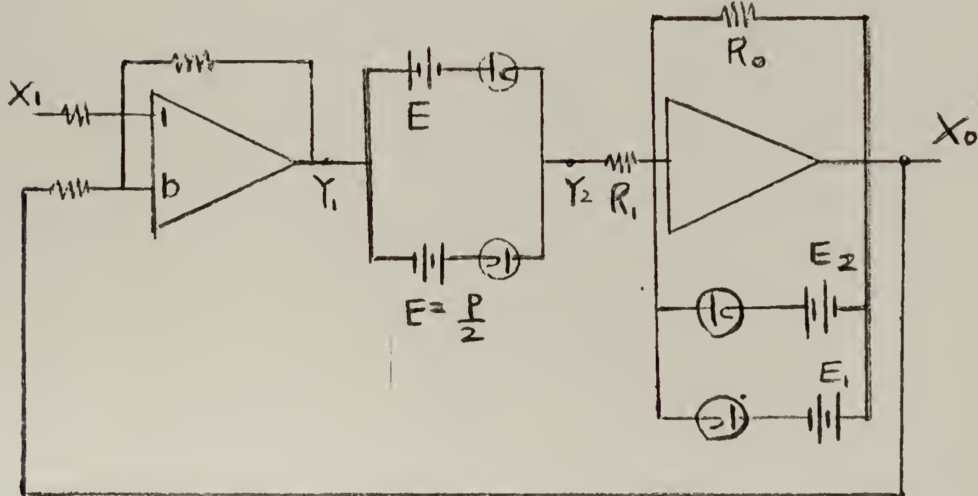


Fig. 7-b Simulation circuit for relay with dead zone and hysteresis

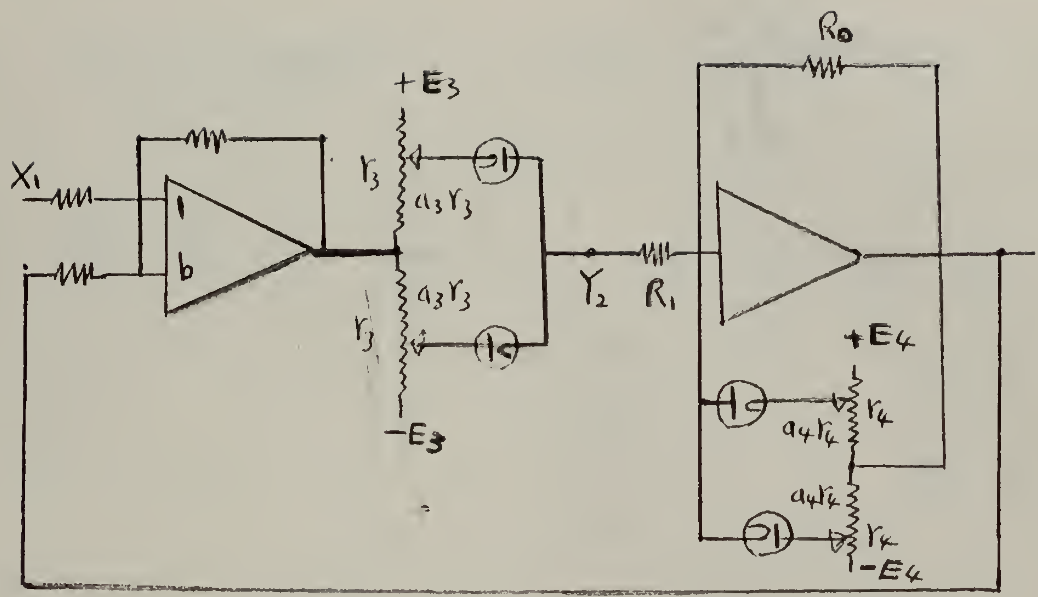


Fig. 7-c Simulation circuit for relay with dead zone and hysteresis

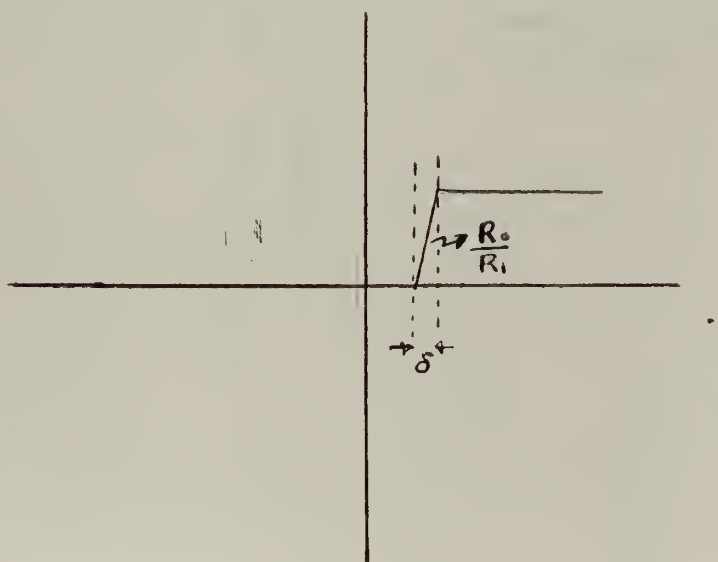


Fig. 7-d Period of unstability of the simulation circuit

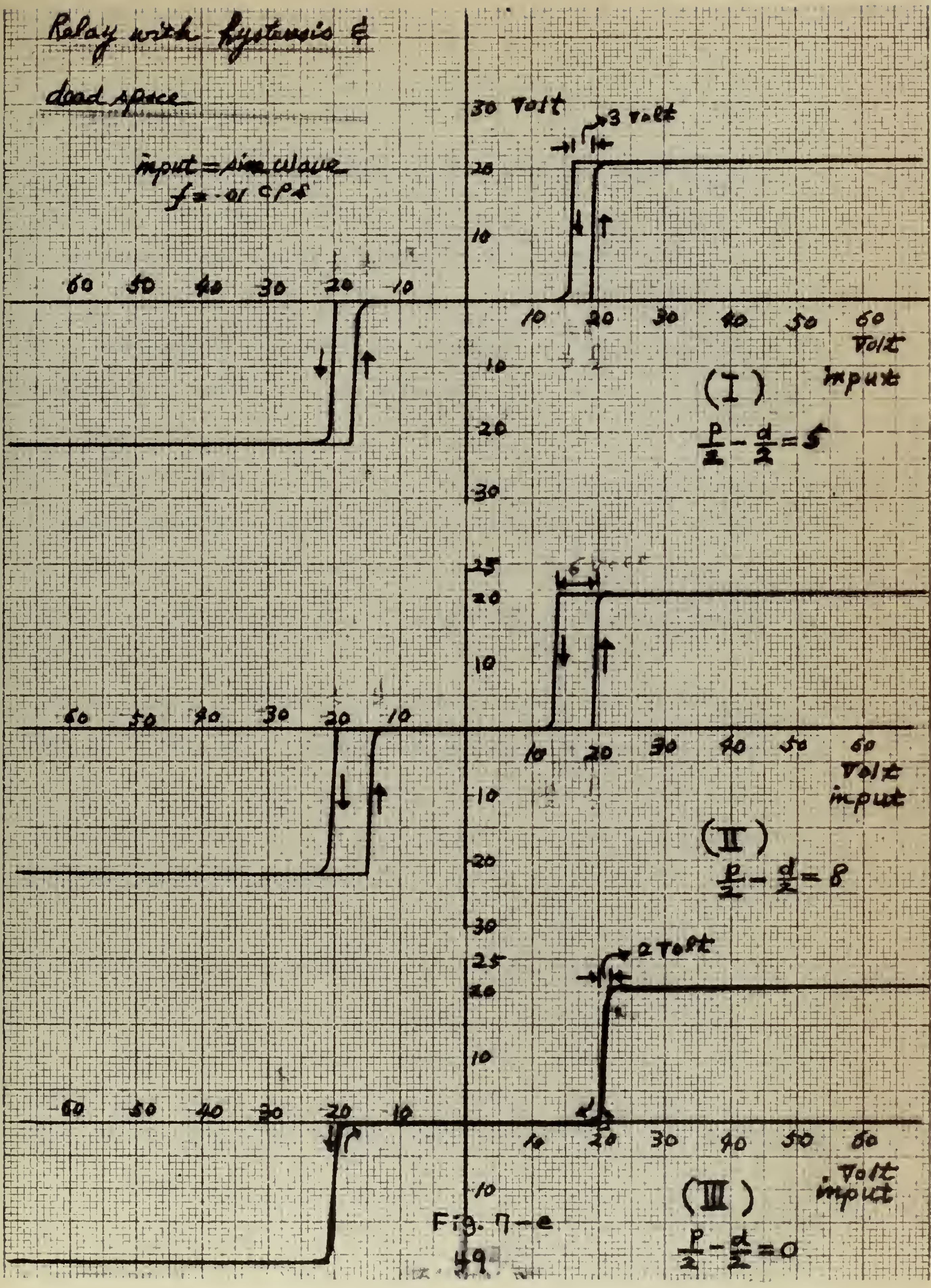

Relay with hysteresis & dead space
input = sine wave
f = .01 cps
30 volt
3 volt
(I)
$\frac{p}{2} - \frac{d}{2} = 5$
volt input
(II)
$\frac{p}{2} - \frac{d}{2} = 8$
2 volt
(III)
$\frac{p}{2} - \frac{d}{2} = 0$

Fig. 7-e

3. Sample simulation of nonlinearities in a simple second order system.

A simple second order linear system of damping factor $= \frac{1}{3}$ is chosen to be simulated with each nonlinearity discussed in section 2.

The method of scaling associated with each nonlinearity is illustrated at each nonlinearity.

Simulation of the linear system is also included in this report for comparison purposes and as a basic linear circuit to be simulated with nonlinearity.

Method of simulation of the linear system used is the transfer function method although the direct component method also is adaptable.

All scaling factors and circuit parameters in the transfer function set up of the linear system are kept consistant throughout all simulation of nonlinearities.

In order for the above stated object, ie., to keep the transfer function of the linear system inside the closed loop to be consistant throughout all nonlinearities simulation, rearrangement in the basic system gain constant was made in the relay servo simulation. These are discussed in detail at paragraph F of section 3.

Also, a step input only was used in this report, and for each nonlinearity, three different plots are obtained. They are:

i. Transient plot of output position and output velocity.

In this plot, two different magnitude of step inputs are used with two different amounts of nonlinearity, and the recording was made through a brush recorder.

ii. Phase plane plot of output position vs. output velocity.

In this plot, two different magnitude steps were used with two different sizes of nonlinearity, same as ones used for plot "a". X-Y plotter was used for plotting.

iii. Frequency response curve.

Two different amplitudes of sine waves were obtained from a function generator and the signal was fed to the simulation board. For each signal, two different size nonlinearities were simulated. In this plot, input and output transient curves were obtained through the brush recorder from which the ratio of output to input magnitude was taken and the curve was plotted on semi log scale graph. The main object of the frequency response curve in this report is just to show that the $M_{p\omega}$ changes in a system if the magnitude of input sine wave is varied due to the presence of nonlinearity in the system. Although a relation between the magnitude of input signal and the frequency response curve could be established in most cases, it is beyond the scope of this report and is omitted.

A. Linear system.

(a) Block diagram (Fig. 1-1)

(b) Mathematical analysis.

$$\ddot{\theta} + \dot{\theta} = K\mathcal{E}$$

$$\frac{\theta_o}{\theta_i} = \frac{2.25}{s^2 + s + 2.25} \triangleq \frac{\omega_n^2}{s^2 + 2\alpha s + \omega_n^2}$$

$$\omega_n = 1.5$$

$$\zeta = \frac{1}{3}$$

$$\omega_n t_p = 2.326$$

$$t_p = 2.22$$

$$M_{pt} = 1.33$$

For step input $\theta_i = r\,u(t)$

$$t_{(\dot{\theta} = max)} = .927 \text{ sec.}$$

$$\dot{\theta}_{o\,max} = .967\, r$$

Let $r \leq .9$

Then

$$t_p = 2.22 \text{ sec.} \qquad \theta_{max} = 1.197$$

$$t_{(\dot{\theta}_{max})} = .927 \qquad \dot{\theta}_{max} = .87$$

(c) Scaling and circuit (Fig. 1-2).

This circuit will be used for all other nonlinearities simulation without changing circuit parameters.

Let:

$$\alpha\,\theta_o \triangleq \frac{\theta_o}{\bar{\theta}_o}$$

$$\alpha\dot{\theta} \triangleq \frac{\dot{\theta}_o}{\bar{\theta}_o}$$

$$\alpha t \triangleq \frac{\bar{t}}{t}$$

And let:

$$\alpha\theta_o = \alpha\,\theta_i = \frac{1.197}{79.7} = .015$$

$$\alpha\dot{\theta} = \frac{.87}{39.6} = .022$$

$$\alpha t = 3$$

$$a_1 = \frac{R_1}{R_{f_1}} = .503 \qquad R_1 = .503\ \mathrm{M.} \qquad R_{f_1} = 1.00\ \mathrm{M}$$

$$a_2 = \frac{R_2}{R_{f_2}} = .5 \qquad R_2 = .500\ \mathrm{M}$$

$$a_3 = (.512)R_3C_{f_3} = .532 \qquad R_3 = 1.00\ \mathrm{M} \qquad C_{f_3} = 1.04\ \mu f$$

$$a_4 = (.488)\ R_4C_{f_4} = .5085 \quad R_4 = 1.00\mathrm{M.} \quad C_{f_4} = 1.04\ \mu f$$

$$a_{o3} = (.333)R_{fo3}C_{f_3} = .3468 \qquad R_{fo3} = 1.00\ \mathrm{M.}$$

(d) Discussion.

For a linear system, the frequency response must be identical for all sizes of input sine waves. The frequency response curves shown on Fig. 1-5 verifies it with little departure. This deficiency was believed to have arisen from reading off the values on the brush recorded curves.

Frequency response data
(linear system)

f (comp.)	$\omega_{(prob)}$	θ_o	M	θ_o	M
.01	.0943	21	1.05	42	1.05
.02	.1885	22	1.1	44	1.125
.03	.283	23.3	1.165	47	1.175
.04	.377	25.5	1.275	51.3	1.285
.05	.472	28	1.4	57.5	1.44
.06	.566	3.	1.55	63	1.575
.07	.66	32.5	1.625	66	1.65
.071	.669	32.6	1.63	65.5	1.635
.072	.678	32.3	1.615	65	1.625
.08	.754	29.5	1.495	60	1.5
.1	.943	19.2	.96	39	.975
.2	1.885	3.4	.17	6.8	.17
.4	3.77	.8	.04	1.5	.0375
1.0	9.43	.11	.0055	.16	.004

$\theta_i = 20 \sin \omega t$ $\qquad$ $\theta_i = 40 \sin \omega t$

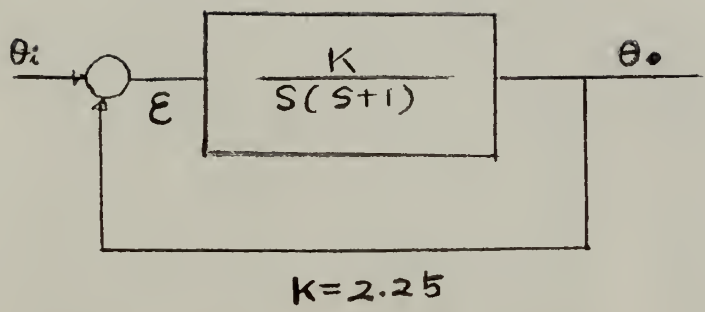


Fig. 1-1 Block diagram of the basic linear system to which various nonlinearities will be added.

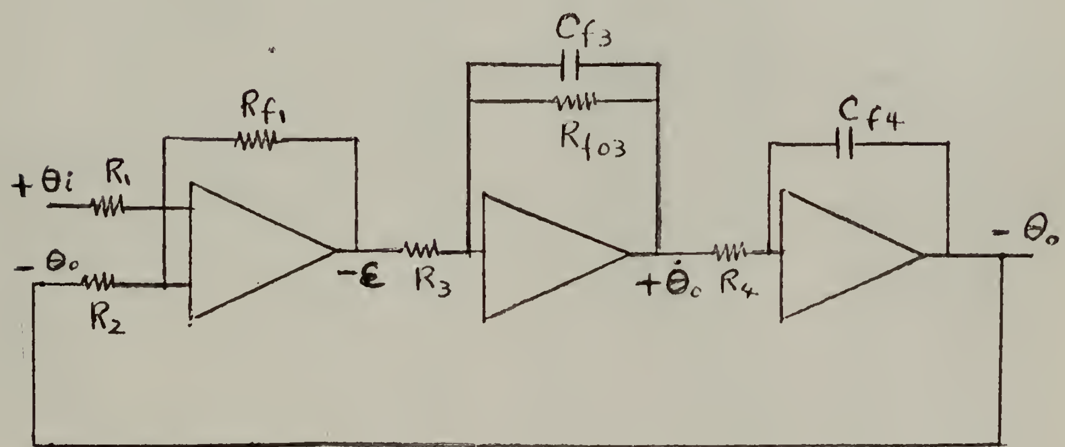


Fig. 1-2 Simulation circuit for the basic linear system.

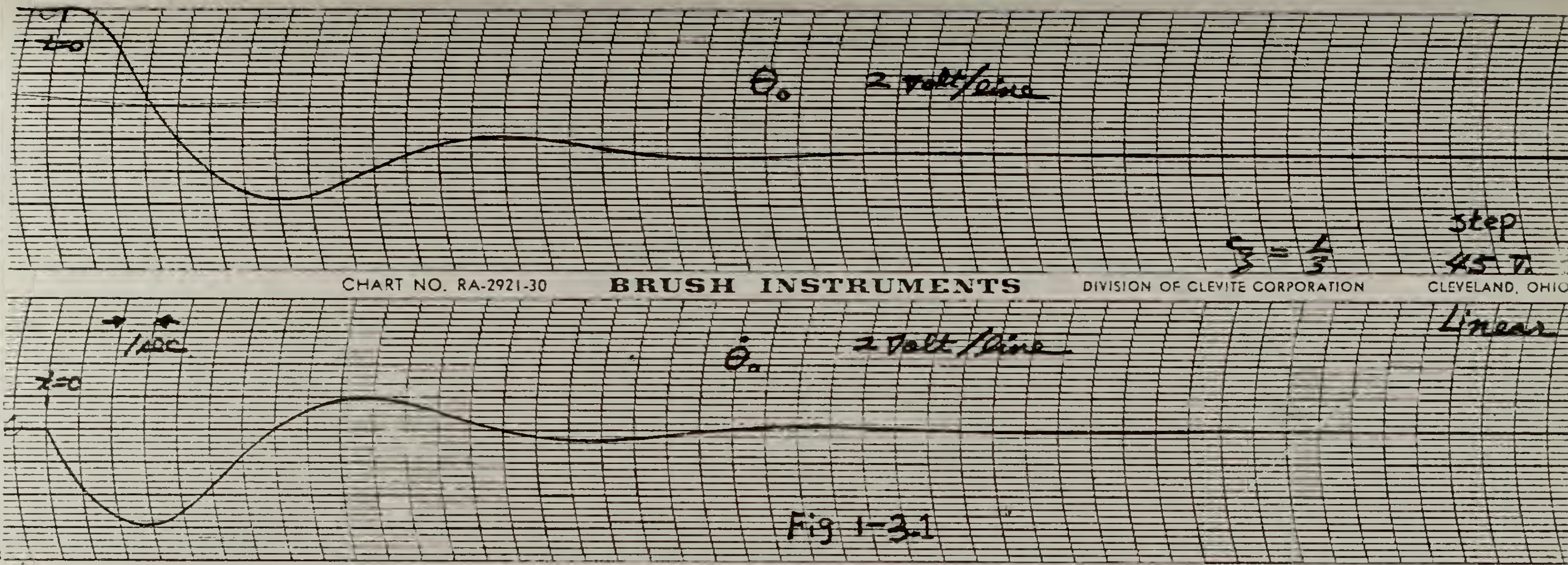


Fig 1-3.1

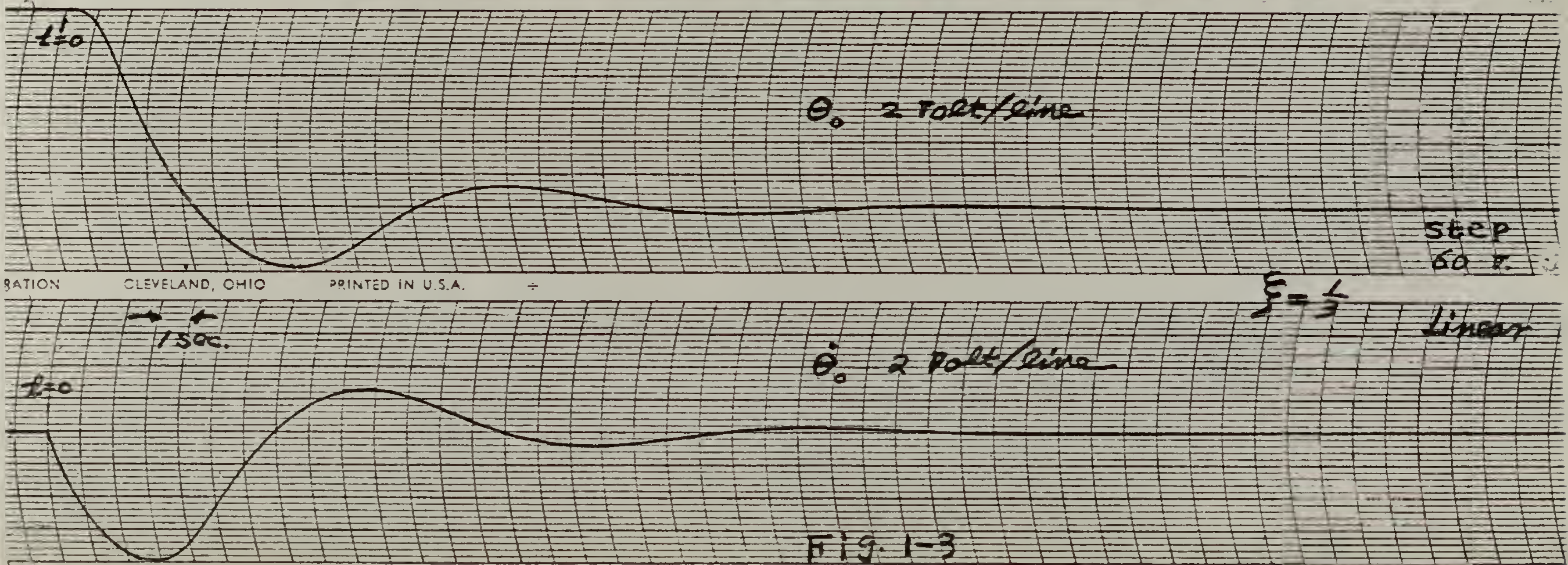


Fig. 1-3

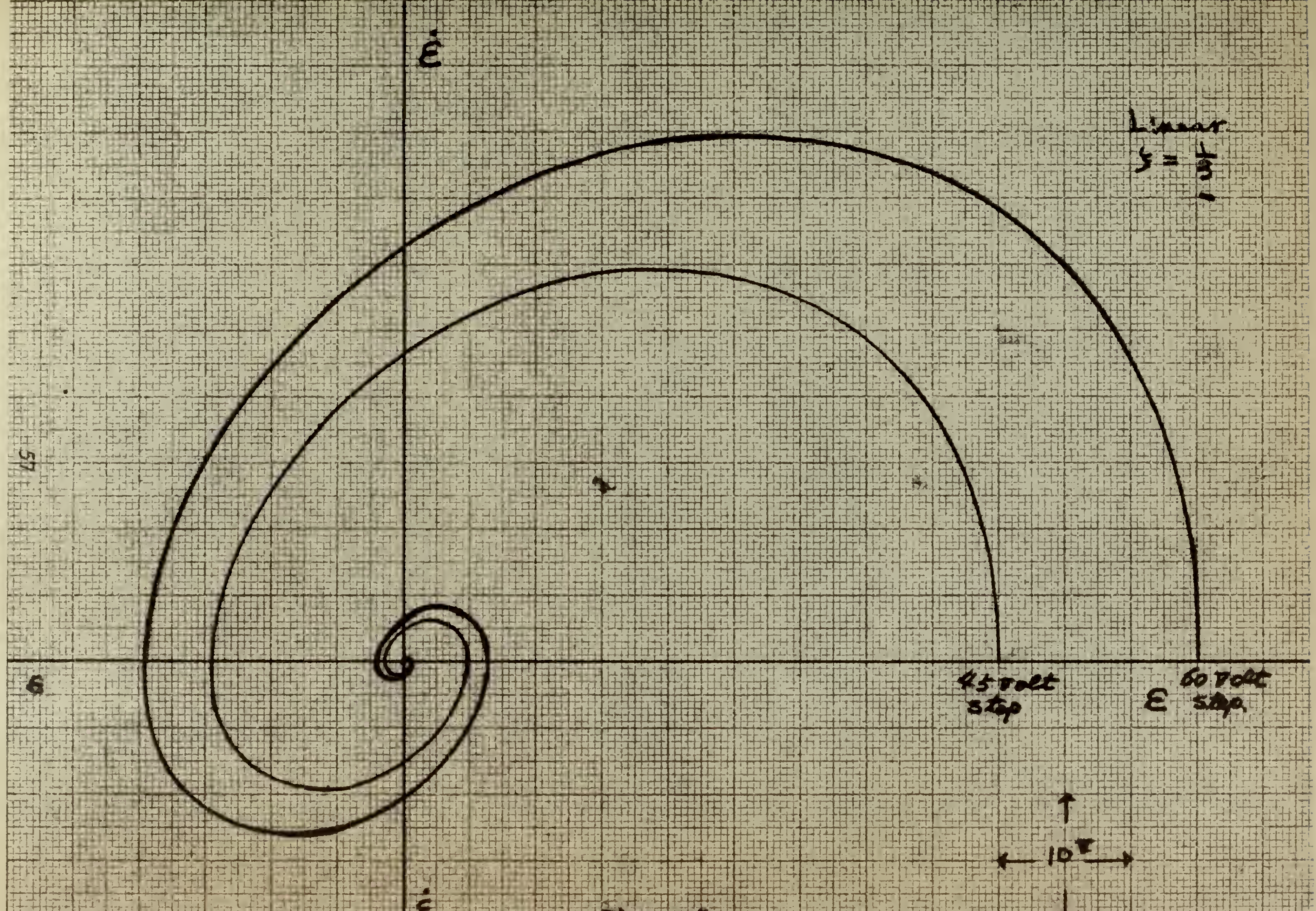


Fig. 1-4

B. Saturation.

(a) Block Diagram.

Saturation in the amplifier can always be referenced to the error signal voltage. If this is done, the circuit to represent the saturation can be placed in the error channel as shown in Fig. 2-1.

(b) Scaling.

Let the amplifier saturate at:

i. $\mathcal{E} = .6$

ii. $\mathcal{E} = .45$

The output of the saturated quantity is still position error. So, the saturated quantity must be scaled the same as positional error.

$$\bar{\mathcal{E}}_1 \, \alpha \mathcal{E}_1 = \mathcal{E}_1$$
$$\bar{\mathcal{E}}_2 \, \alpha \mathcal{E}_2 = \mathcal{E}_2$$

Let:

$$\alpha \mathcal{E}_1 = \alpha \mathcal{E}_2 = \alpha \theta = .015$$

$$\bar{\mathcal{E}}_1 = \frac{.6}{.015} = 40 \text{ v.}$$

$$\bar{\mathcal{E}}_2 = \frac{.45}{.015} = 30 \text{ v.}$$

(c) Simulation circuit (Fig. 2-2)

Where:

$$R_1 = R_2 = R_3 = R_4 = 1.00 \text{ M.}$$

$$r = .1 \text{ M}$$

$$a_1 = a_2 = .333$$

$$E = \begin{matrix} i & 80 \text{ v.} \\ ii & 60 \text{ v.} \end{matrix} \qquad \frac{a_1}{1-a_1} E = 40, \quad \frac{a_2}{1-a_2} E = 30 \text{ v.}$$

$$\frac{a_1 r_1}{R_1} = .0333$$

(d) Resultant curves.

Fig. 2-3 and 2-4 shows transient responses of the system to 45 volt and 60 volt step inputs respectively. The lower the saturation voltage, the lower the overshoot and the lesser the output velocity. These are well shown in both transient and phase plane curves.

If the input signal increases, saturation plays its part for a longer period and the above mentioned effects become pronounced, as is shown in Fig. 2-4.

In the frequency response curve, saturation decreases $M_{p\omega}$ by comparison with the linear system and the saturation effect is greater for a larger input signal in identical system, thus decreasing the $M_{p\omega}$. This situation conforms to the established theory.

(e) Discussion.

i. Error which may have been introduced due to the insertion of a saturation circuit will be due to the term $\frac{a_1 r_1}{R_1} = .0333$.

Roughly this will give 3% of error in error signal. This error can be made very small either by using smaller resistance potentiometer or by chosing larger voltage sources.

Frequency response data
(saturation)

$T(sat.) = 30\ T$

f (comp)	w (prob)	$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	20	1.00	40.8	1.02
.02	.1885	20.7	1.035	42.2	1.055
.03	.283	22	1.1	45	1.125
.04	.377	24	1.2	49	1.225
.05	.472	27	1.35	55	1.375
.052	.481	27.5	1.375	55	1.375
.058	.547	29.5	1.475		
.06	.566	30	1.5	45.5	1.138
.065	.613	30.5	1.525		
.07	.66	30	1.5	35.5	.888
.1	.943	16	.8	18	.45
.2	1.885	3.5	.175	4.5	.1125
.5	4.72	.5	.025	.75	.0187
1.0	9.43	.1	.005	.09	.00225

$\bar{\theta}_i = 20 \sin \omega t$ $\qquad$ $\bar{\theta}_i = 40 \sin \omega t$

Frequency response data
(saturation)

$V_{(sat.)} = 40\ v.$

f (comp)	ω (prob)	$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	20	1.00	41.8	1.045
.02	.1885	20.7	1.035	42.8	1.07
.03	.283	22.2	1.11	45	1.125
.04	.377	24.5	1.225	49.5	1.235
.05	.472	27	1.35	55	1.375
.056	.528			59	1.475
.058	.547			59.5	1.485
.06	.566	30	1.5	58.5	1.465
.065	.613	31	1.55	53	1.325
.068	.642	31.1	1.555		
.07	.66	31.1	1.555		
.08	.754	28.7	1.435	32.5	.813
.1	.943	18	.9	24.5	.613
.2	1.885	2.6	.13	6.2	.155
.5	4.72	.45	.0225	1.0	.025
1.0	9.43	.1	.005	.2	.005
		$\bar{\theta}_i = 20 \sin \omega t$		$\bar{\theta}_i = 40 \sin \omega t$	

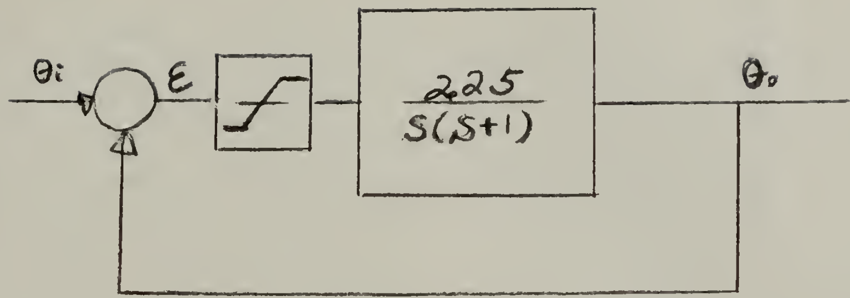


Fig. 2-1 Block diagram of the system with saturation.

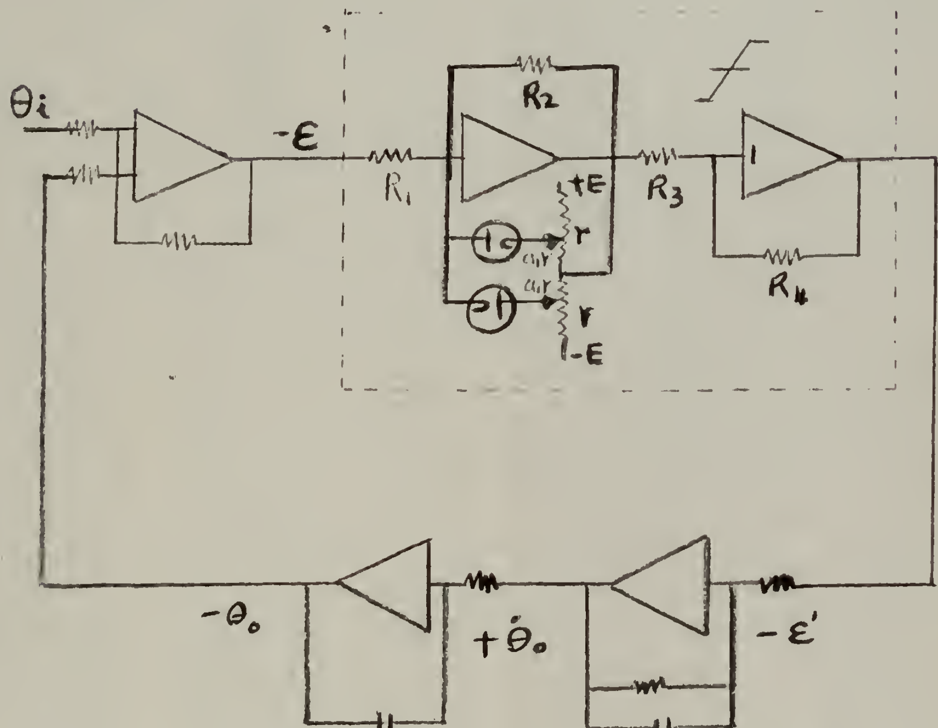


Fig. 2-2 Simulation circuit for the system shown in Fig.2-1.

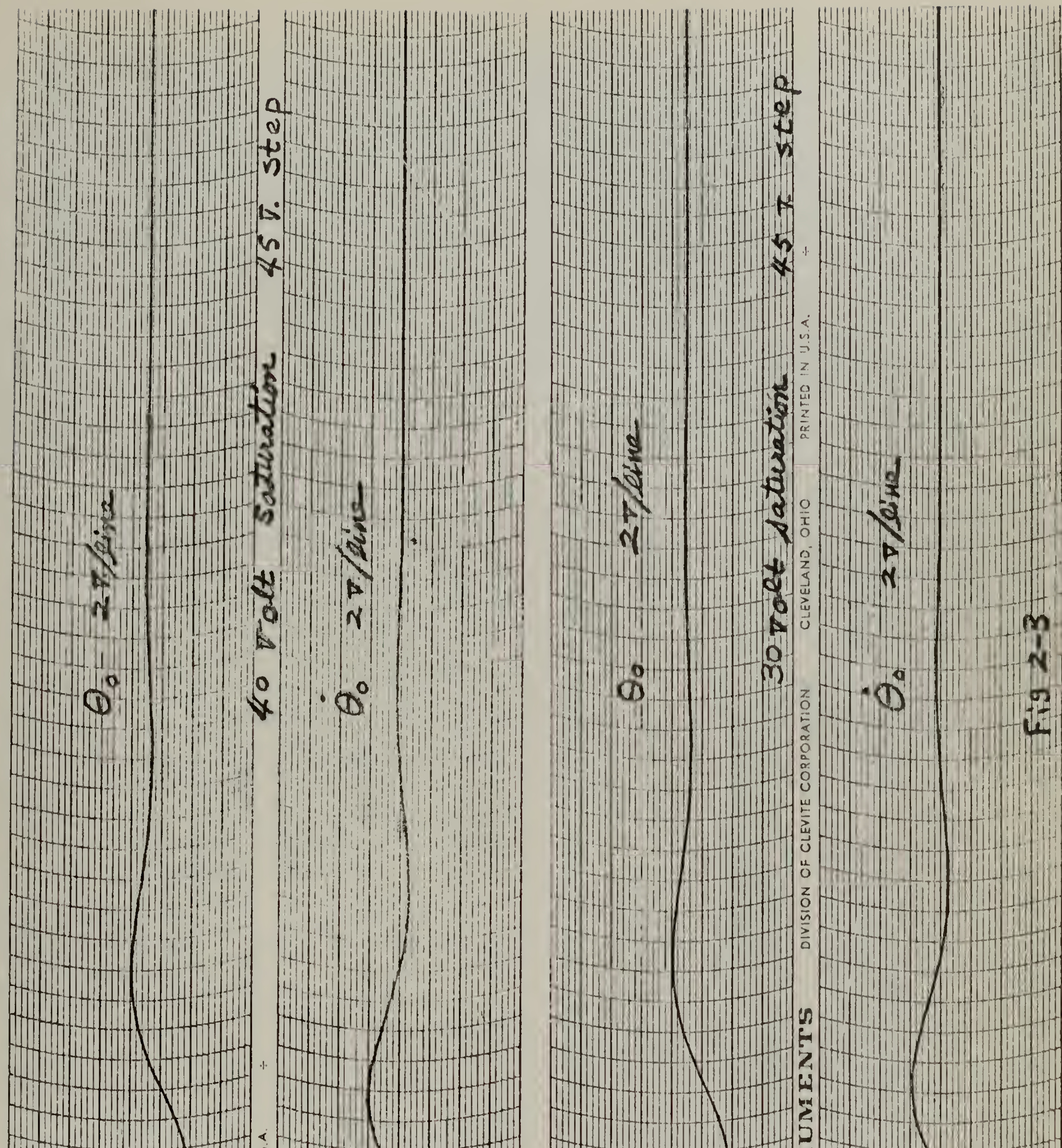
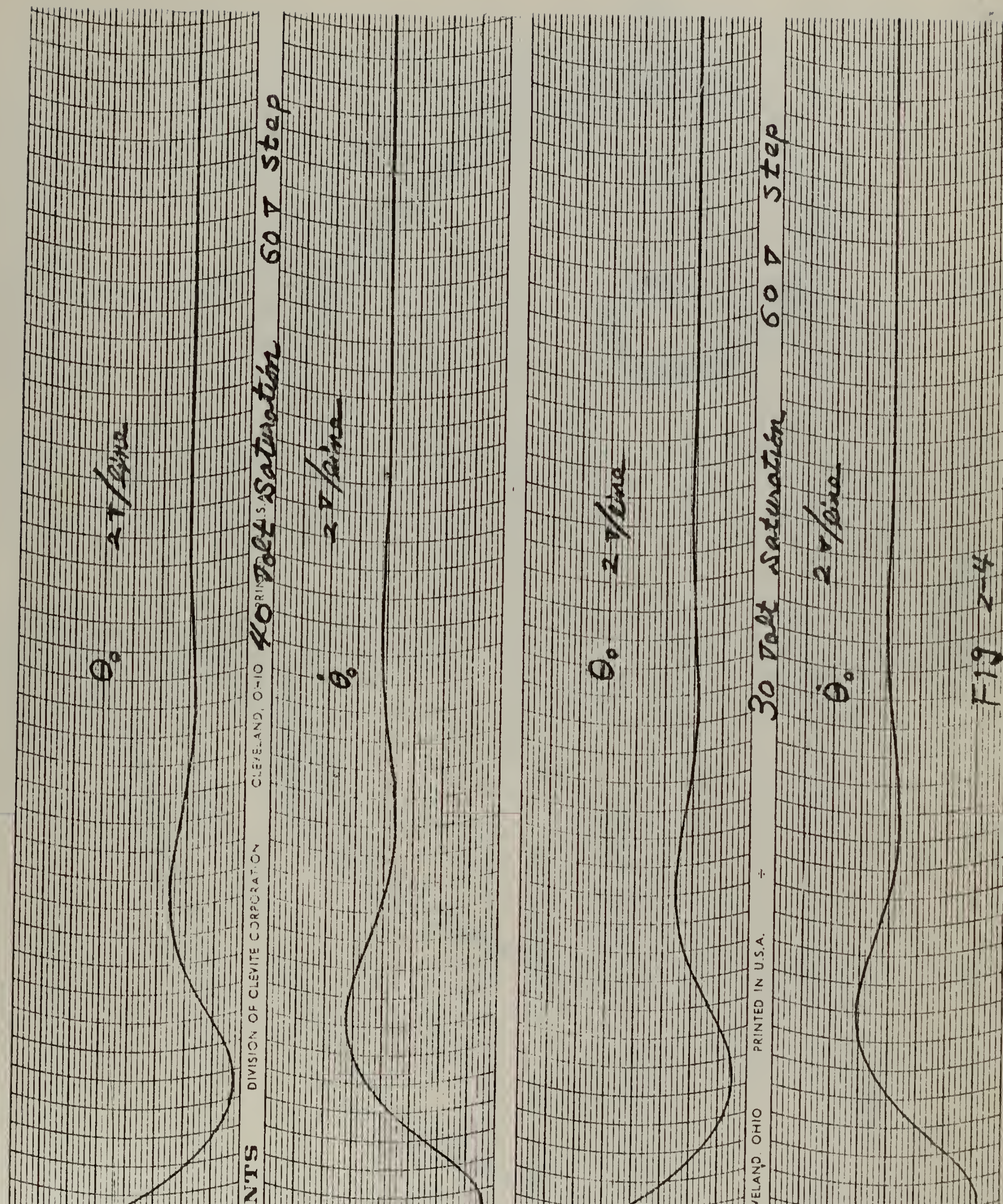
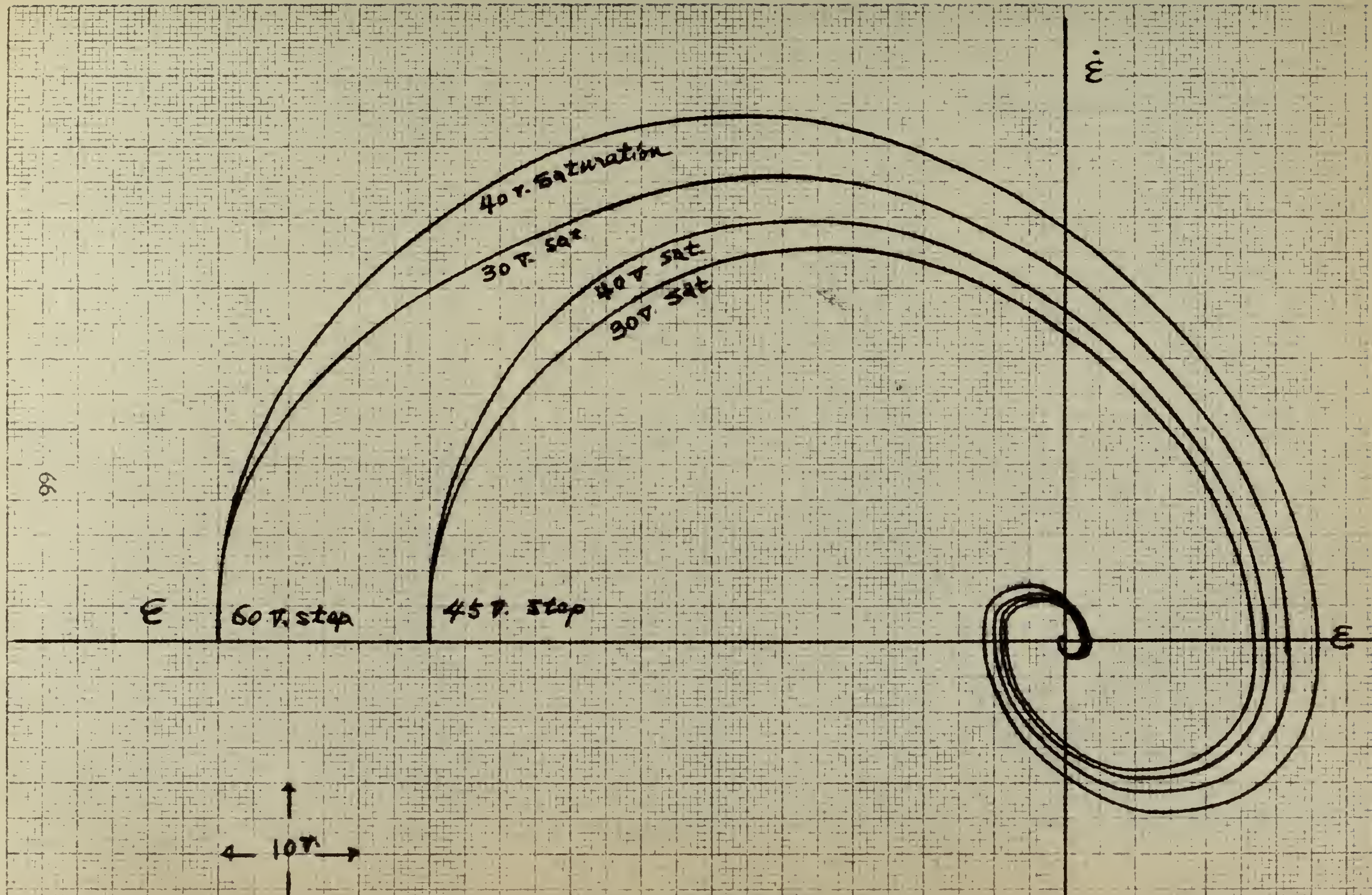

θ_o 2v/div
40 volt Saturation 45 V. step
$\dot{\theta}_o$ 2v/div
$t = 0$
1/sec
θ_o 2v/div
30 volt saturation 45 V. step
$\dot{\theta}_o$ 2v/div
1/sec
BUSH INSTRUMENTS
DIVISION OF CLEVITE CORPORATION CLEVELAND, OHIO PRINTED IN U.S.A.

Fig 2-3



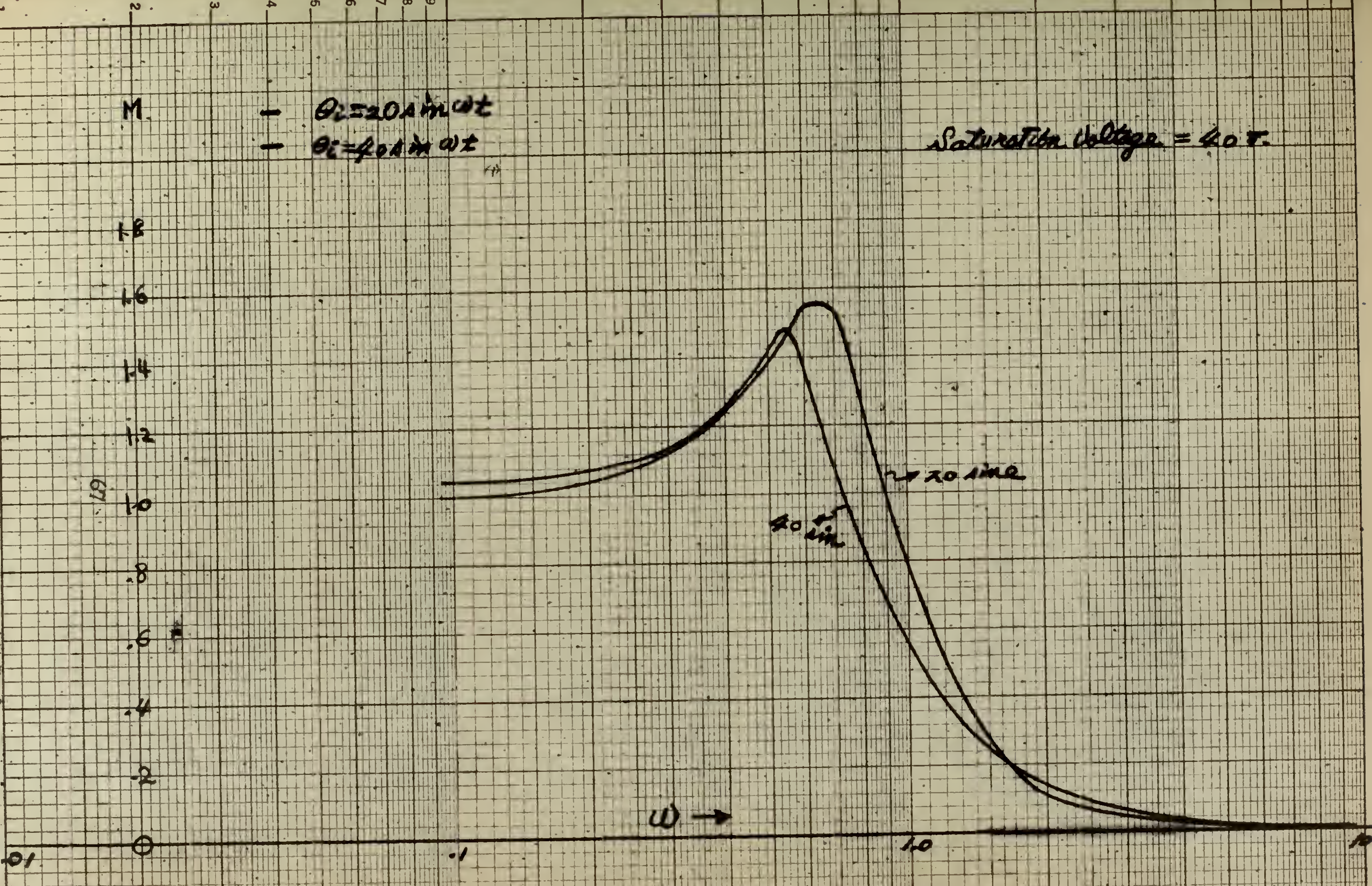
θo 2 v/line
40 Volt Saturation 60 v step
θ̇o 2 v/line
1 sec
θo 2 v/line
30 Volt Saturation 60 v step
θ̇o 2 v/line
1 sec
DIVISION OF CLEVITE CORPORATION
CLEVELAND, OHIO
PRINTED IN U.S.A.

Fig 2-4



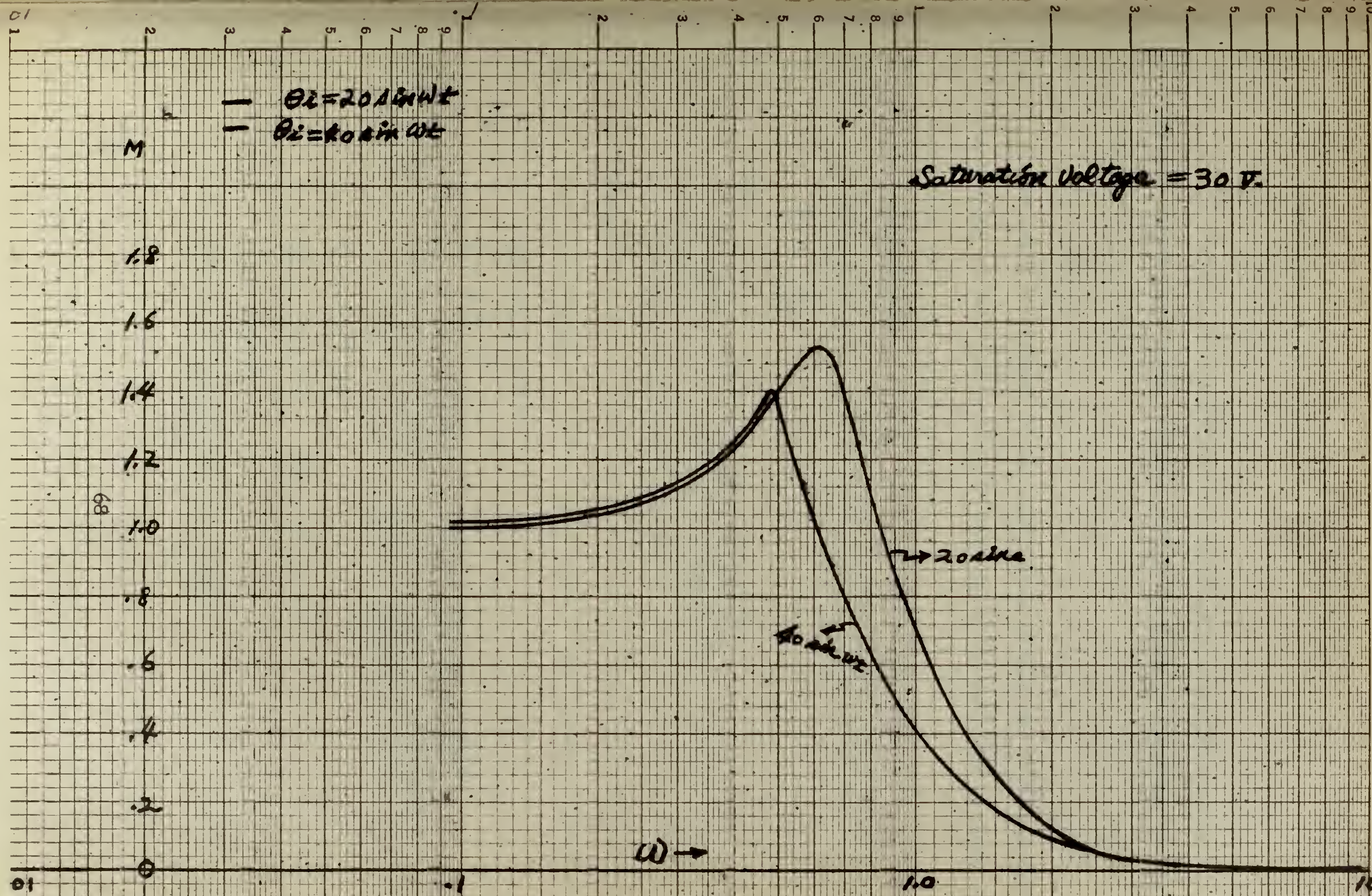
Saturation

Fig. 2-5



Saturation

Fig. 2-6



Saturation

Fig. 2-9

C. Dead space or Free play.

(a) Block diagram.

A dead space is assumed to be present about the zero position in the spring loaded load shaft, and the output position is measured at the load shaft (Fig. 3-1).

(b) Scaling.

The input and output of the dead space circuit are output position of the system, so that the scaling of dead space circuit must be done in accordance with the output position.

$$D = \bar{D}\,\alpha D$$

$$\alpha D \triangleq \alpha\,\theta$$

$$\bar{D} = \frac{D}{\alpha\theta}$$

Let:

i $D = .05$ $\qquad \bar{D} = 3.33\ V.$

ii $D = .1$ $\qquad \bar{D} = 6.667\ V.$

(c) Simulation circuit (Fig. 3-2)

Where:

$r = 100\,K$

$R_1 = R_0 = 1.00\,M$

$E = 45\ V$

a_1 i $a_1 = .0357$ $\qquad \frac{\bar{D}}{2} = \frac{a_1}{1-a_1}E = \frac{3.33}{2}$

ii $a_1 = .0689$ $\qquad \frac{\bar{D}}{2} = \frac{a_1}{1-a_1}E = \frac{6.667}{2}$

$C = 1.09\ \mu f$

$R_2 = .02\,M \qquad R_3 = 5.02\,M$

$a_2 = .374$

$$\bar{\theta}_o = -\frac{R_3 C S}{1 + R_2 C S}\,a_2\,\bar{\theta}_o$$

Since: $R_2 C \ll 1 \qquad \bar{\dot{\theta}} \approx -a_2 R_3 C S \bar{\theta}_0$

But
$$\bar{\dot{\theta}}\,\alpha\dot{\theta}_0 = \alpha\theta\,\alpha t\,(S\bar{\theta}_0)$$
$$\because \alpha t \triangleq \frac{T}{t} \qquad d\,t_{(p)} = \frac{1}{\alpha t}\,d\,t_{(c)}$$
$$S = \frac{d}{dt} = \alpha t\,\frac{d}{d(t_c)}$$
$$\bar{\dot{\theta}}_0 = \frac{\alpha\theta_0}{\alpha\dot{\theta}_0}\,\alpha t\,S\,\bar{\theta}_0$$
$$\therefore a_2 R_3 C = \frac{\alpha\theta_0\,\alpha t}{\alpha\dot{\theta}_0}$$
$$a_2 = .374$$

(d) Resultant curves.

Since the dead space was assumed to be present about the zero position of spring loaded load shaft, it plays little effect on the system response for step inputs of the magnitude selected.

The effect is almost indistinguishable on transient response curves.

In the phase plane plot, it is better shown. If the size of dead zone becomes higher, output velocity at the start becomes higher. After some time has passed, the phase plane becomes almost same for different size of dead zone.

However, it has a clear effect on the frequency response curves showing definitely different responses to different sizes of the input signal, thus indicating a definite presence of nonlinearity in the system.

(e) Discussion.

The slope of input vs. output of nonlinearity simulation circuit is:
$$\frac{R_0}{R_1}\;\frac{1}{1 + \frac{a_1}{1-a_1}\,\frac{a_1 r}{R_1}}$$
as discussed on paragraph B of section 2.

However, the potentiometer loading was corrected in this case, then;

$$\text{slope} = \frac{R_0}{R_1}\frac{1}{\frac{1}{1-a_1}} = \frac{R_0}{R_1}(1-a_1) = .9643 \quad \text{for} \quad a_1 = .0357$$

$$= .9311 \quad \text{for} \quad a_1 = .0689$$

The % error is about three to seven %. But it can be reduced by adjusting $\frac{R_0}{R_1}$ so that $\frac{R_0}{R_1}(1-a_1) \triangleq$ slope.

Frequency response data
(dead space)

$\bar{D} = 3.33$

		$\bar{\theta}_i = 20 \sin \omega t$		$\bar{\theta}_i = 40 \sin \omega t$	
$\bar{f}$	ω	$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	20.2	1.01	40.8	1.022
.02	.1885	20.7	1.035	42	1.05
.03	.283	23	1.15	44	1.1
.04	.377	25.5	1.275	48.5	1.212
.05	.472	28.4	1.42	54	1.35
.06	.566	30.3	1.515	58	1.45
.065	.613	30.7	1.535	59	1.475
.07	.66	29.7	1.485	58	1.45
.08	.754	25.2	1.26	51.5	1.289
.09	.843	19.5	.975	41.5	1.039
.1	.943	14.1	.705	32	.8
.2	1.885	1.65	.0825	4.5	.1125
.4	3.77	.36	.018	.43	.01075
.6	5.66	.18	.009	.3	.0075
.8	7.54	.08	.004	.2	.005
1.0	9.43	.04	.002	.006	.0015

Frequency response data
(dead space)

$\bar{D} = 6.667$

		$\bar{\theta}_i = 20 \sin \omega t$		$\bar{\theta}_i = 40 \sin \omega t$	
$\bar{f}$	ω	$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	20.3	1.015	40.5	1.012
.02	.1885	21.3	42	1.05	
.03	.283	23.5	1.175	45.5	1.088
.04	.377	26	1.3	49.5	1.237
.05	.472	28.4	1.42	55	1.375
.06	.566	29.8	1.49	58.5	1.463
.065	.613	29.1	1.453	58	1.45
.07	.66	27.2	1.36	55.5	1.387
.08	.754	21.8	1.09	47	1.175
.09	.848	15.8	.79	37	.925
.1	.943	11	.55	28	.70
.2	1.885	.65	.0325	2.7	.0675
.4	3.77	.35	.0175	.4	.01
.6	5.66	.17	.0085	.24	.006
.8	7.54	.06	.003	.15	.00375
1.0	9.43	.04	.002	.1	.0025

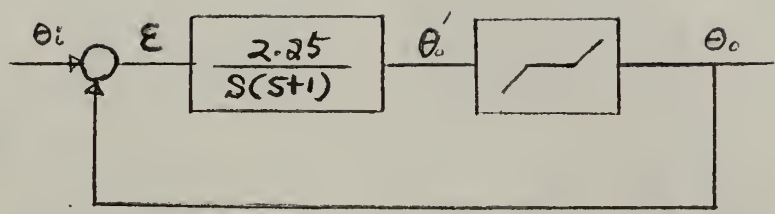


Fig. 3-1   Block diagram of the system with dead space.

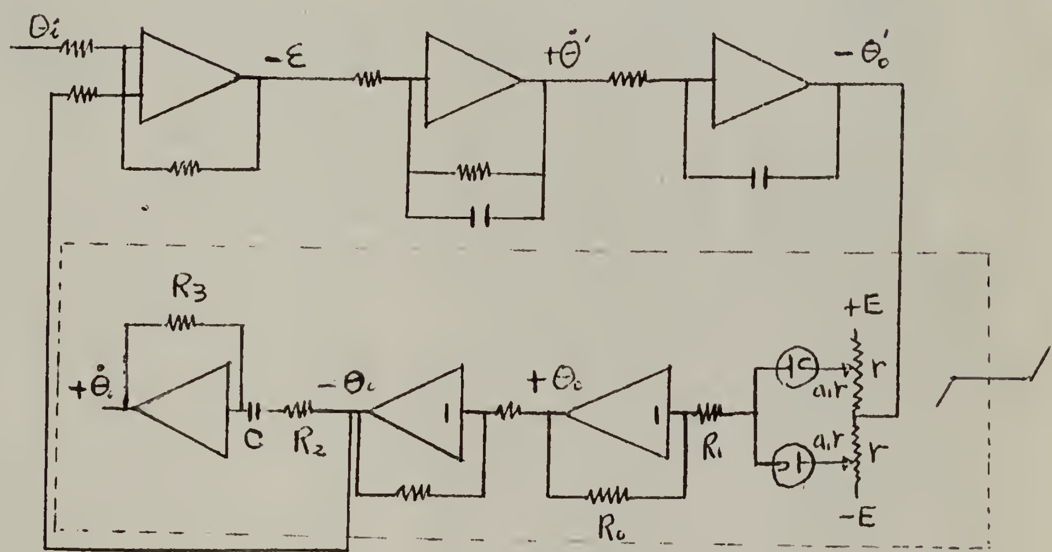


Fig. 3-2   Simulation circuit for the system shown in Fig. 3-1.

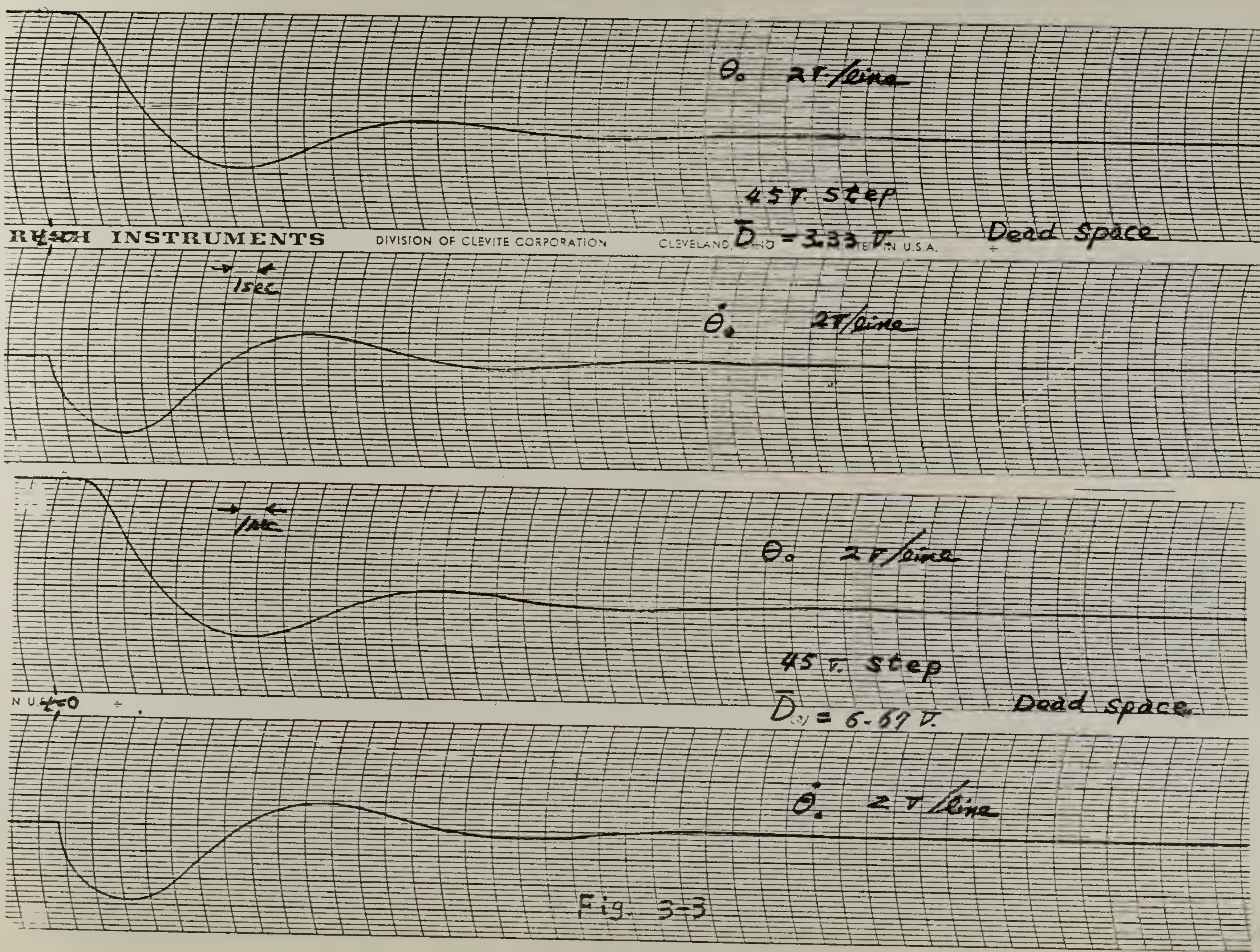

θ₀ 2V/line
45V step
$\bar{D}$ = 3.33 V.
Dead Space
RUSH INSTRUMENTS DIVISION OF CLEVITE CORPORATION CLEVELAND, OHIO PRINTED IN U.S.A.
1sec
$\dot{\theta}_0$ 2V/line
1sec
θ₀ 2V/line
45V. step
$\bar{D}$ = 6.67 V.
Dead space
$\dot{\theta}_0$ 2V/line

Fig. 3-3

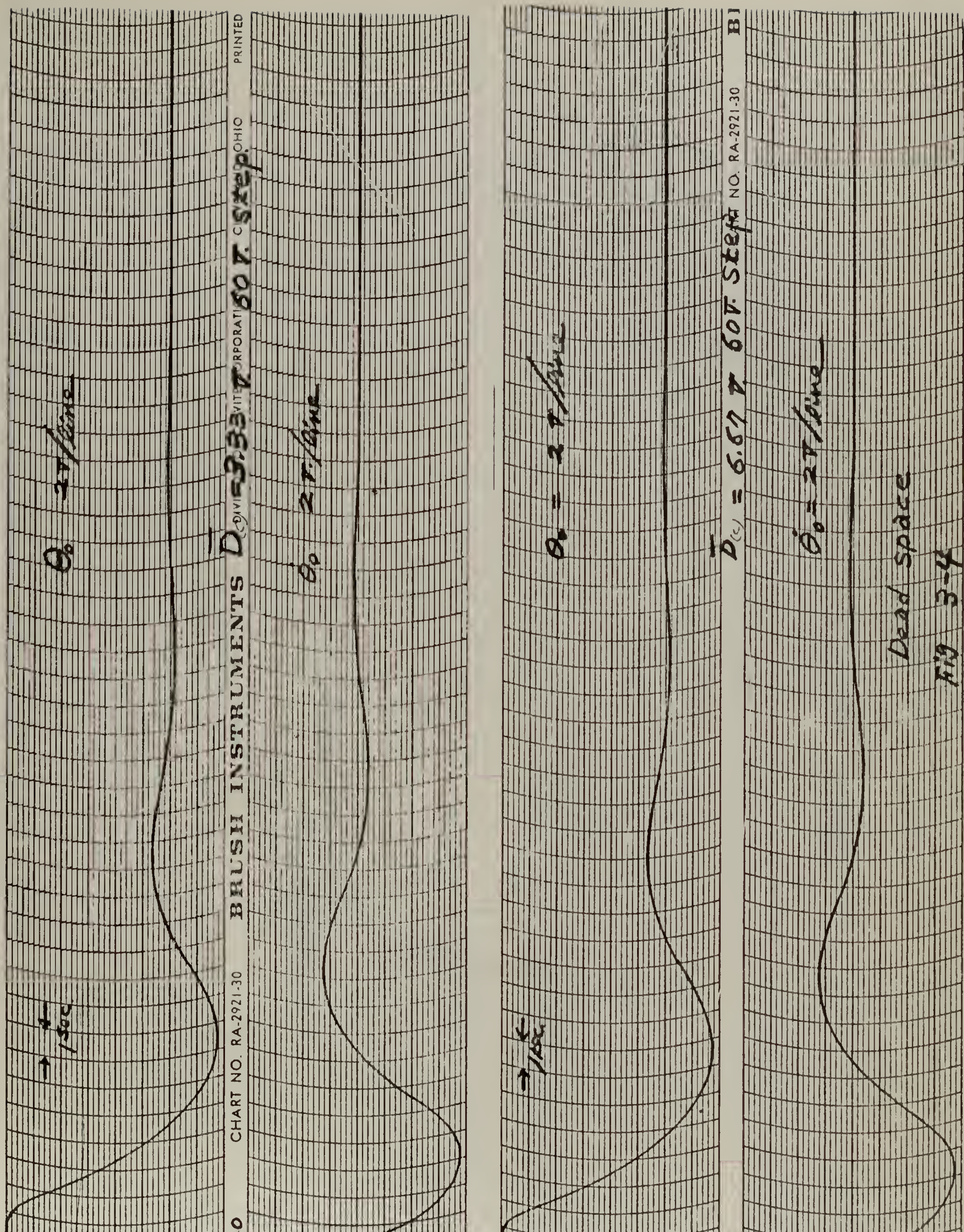
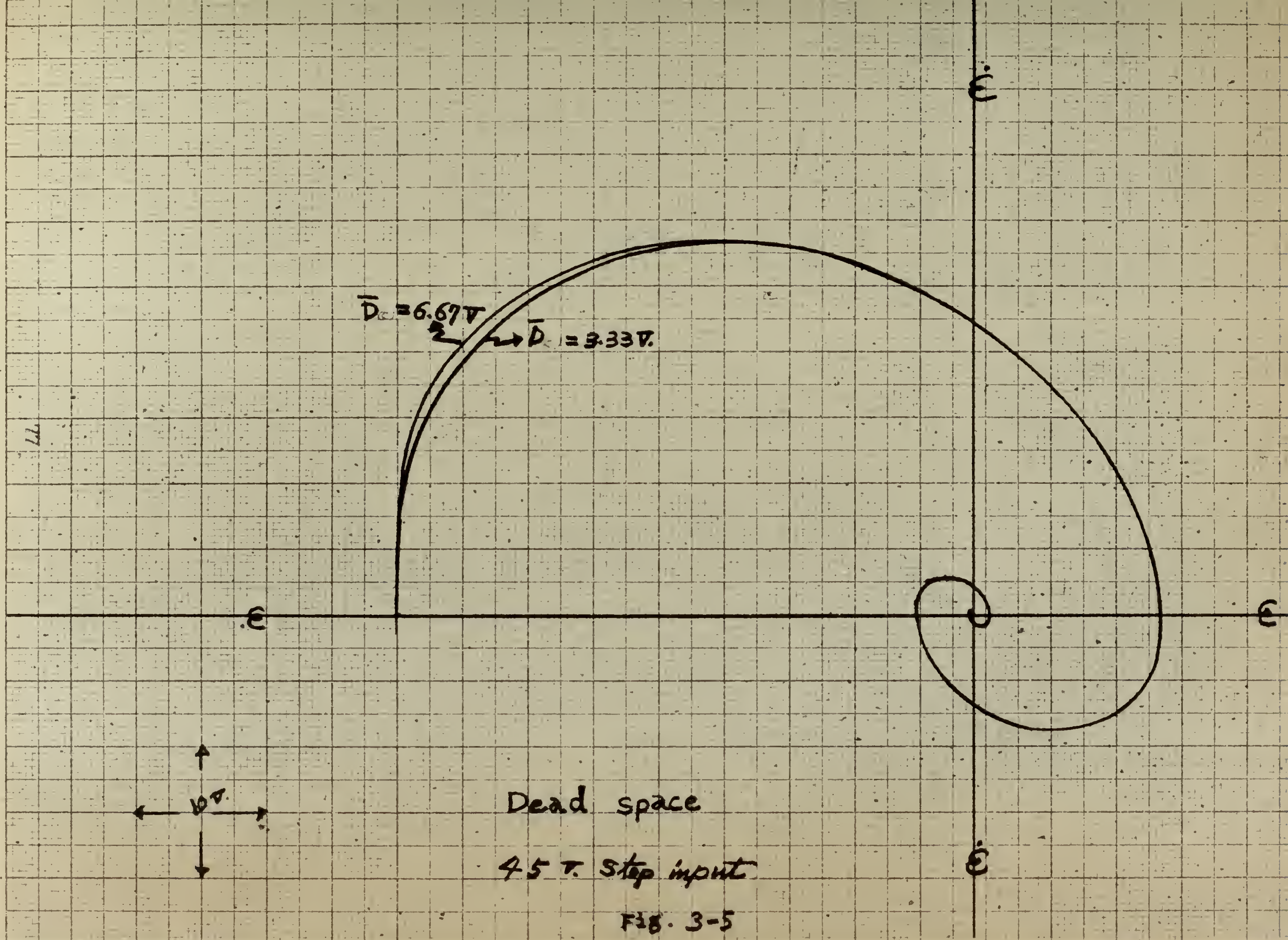


CHART NO. RA-2921-30
BRUSH INSTRUMENTS
DIVISION
CORPORATION
CLEVELAND, OHIO
PRINTED
θ_0 2°/line
$\bar{D}_{(c)}$ = 3.33° 60° step
$\dot{\theta}_0$ 2°/line
1 sec
t=0
θ_0 = 2°/line
$\bar{D}_{(c)}$ = 6.67° 60° step
$\dot{\theta}_0$ = 2°/line
1 sec
t=0
Dead Space

Fig 3-4



Dead space

45 v. step input

Fig. 3-5

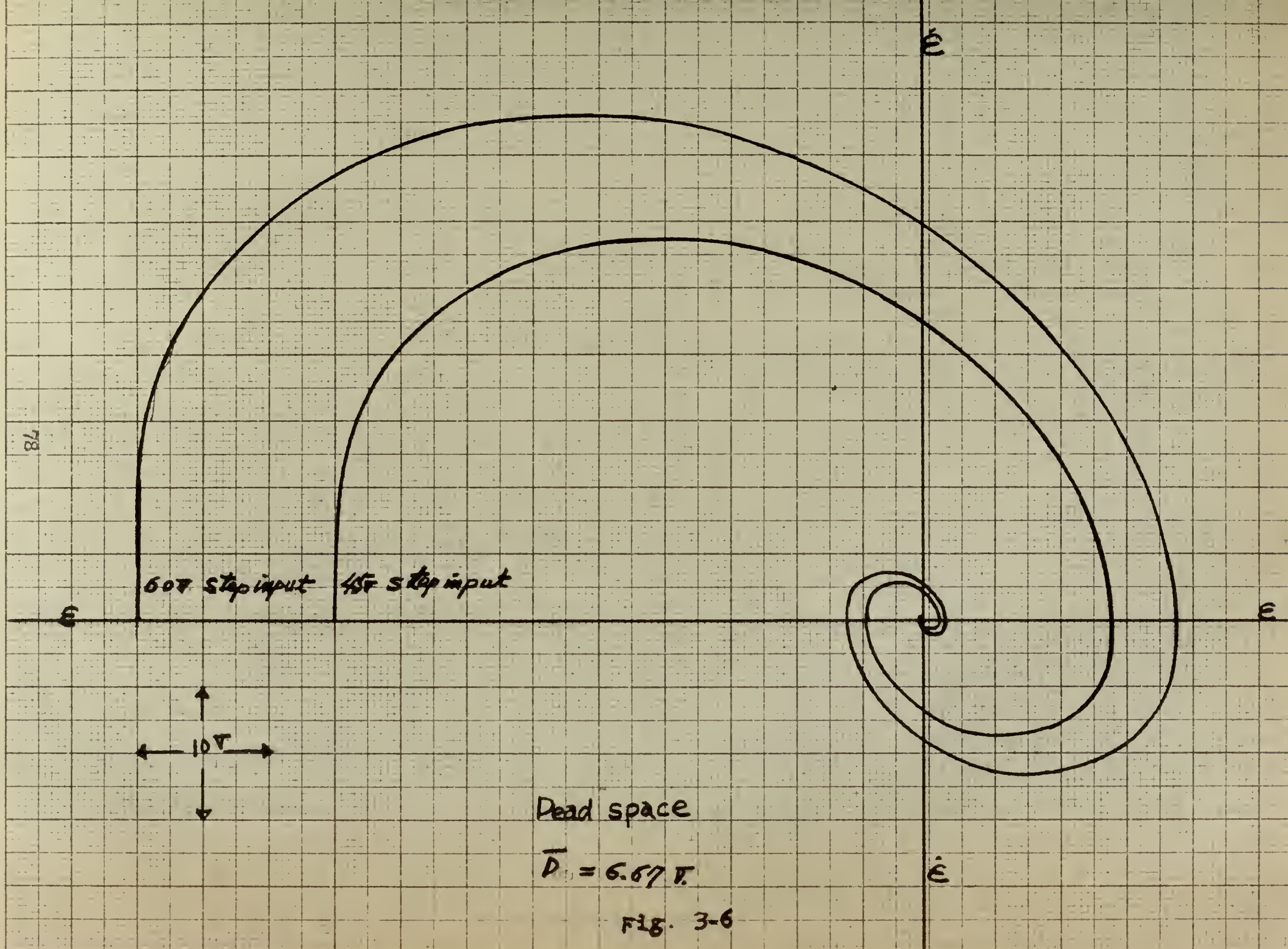


Fig. 3-6

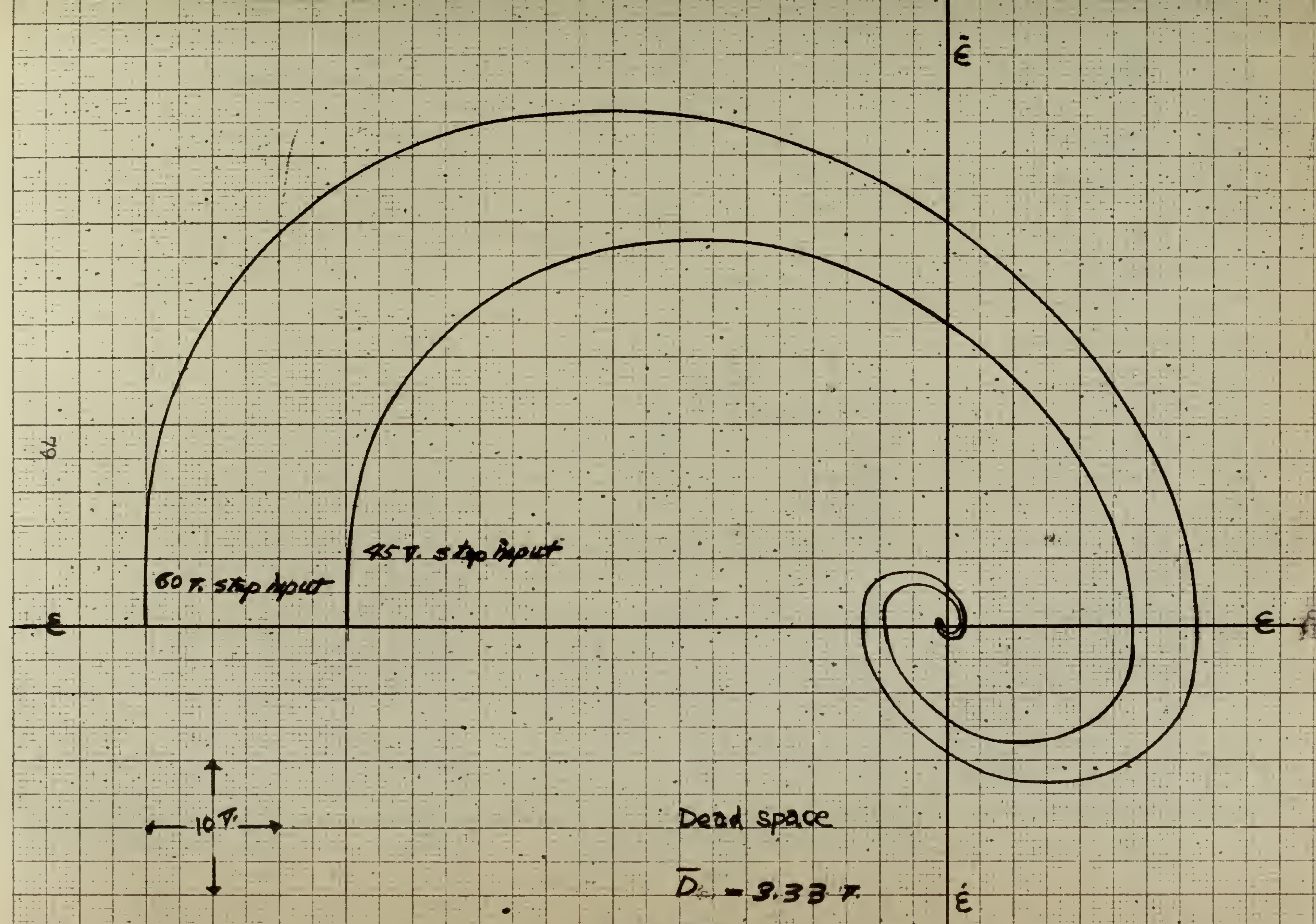


Fig. 3-7

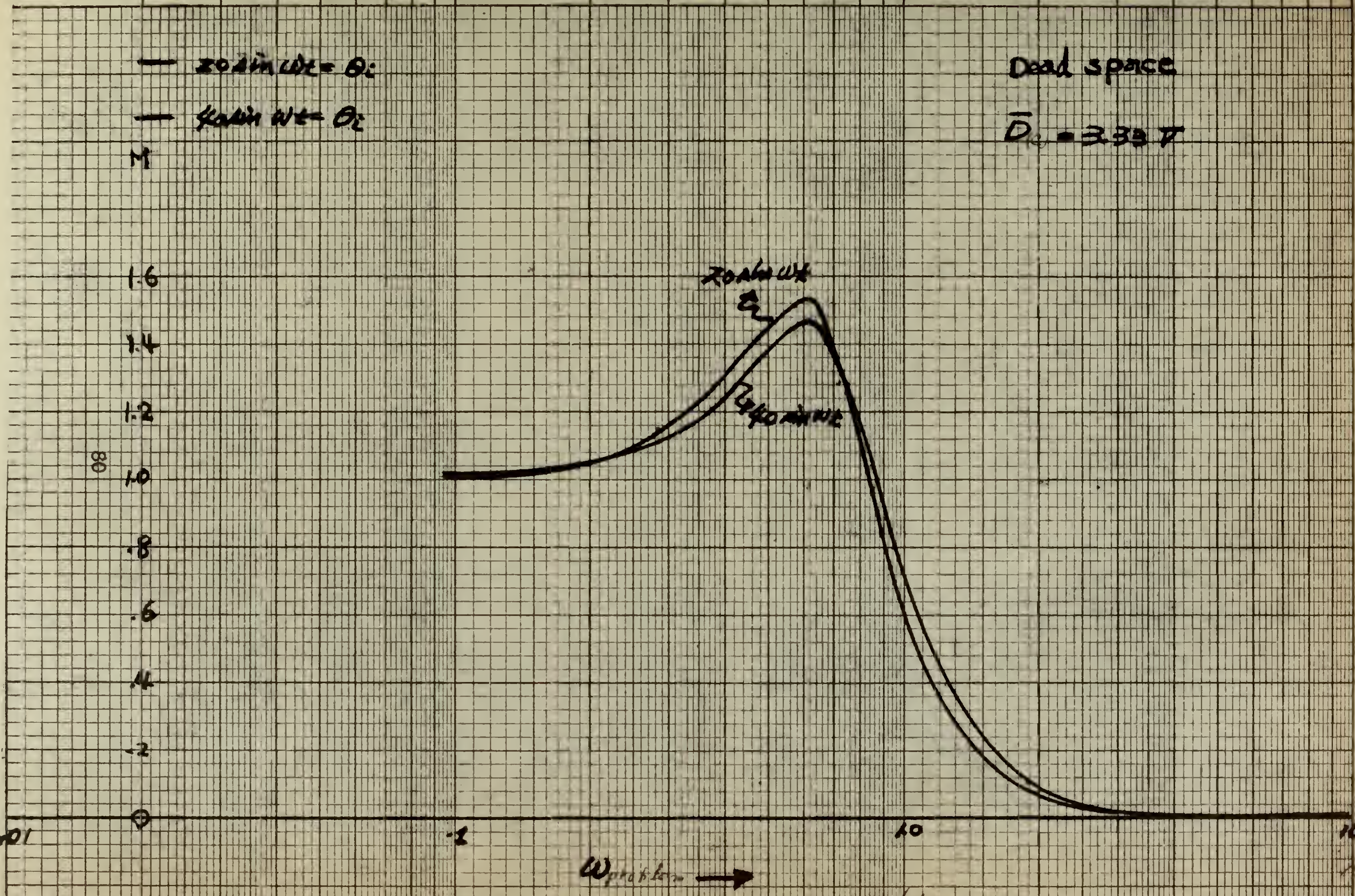


FIG. 3-8

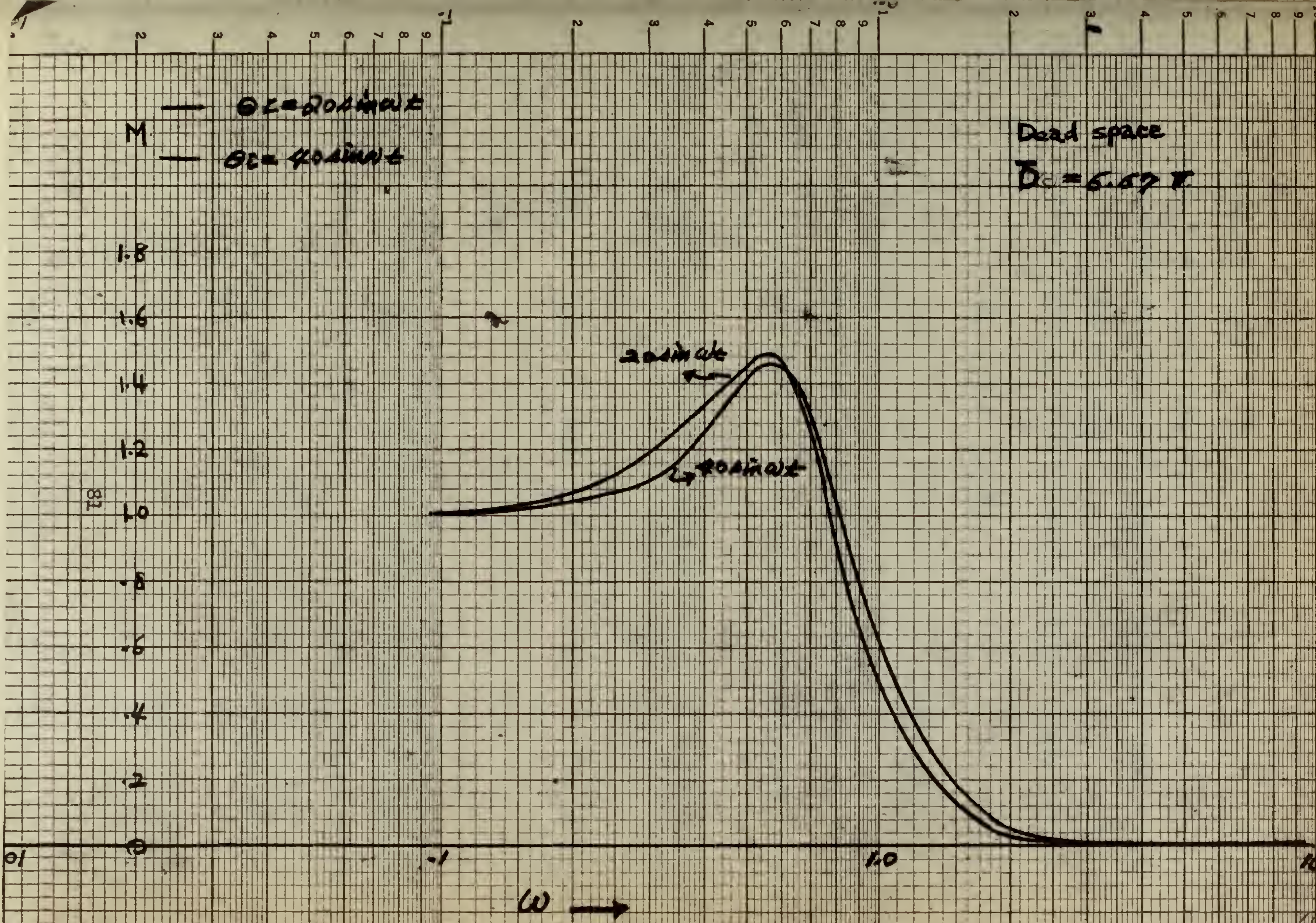


FIG. 3-9

D. Coulomb friction.

(a) Block diagram (Fig. 4-1).

The system open loop transfer function F(s) was chosen to be:

$$F(s) = \frac{1}{1+s} \qquad 4.1$$

From this equation, the system equation without coulomb friction becomes:

$$\ddot{\theta}_o + \frac{f}{J}\dot{\theta}_o = K\varepsilon \qquad 4.2$$

If coulomb friction C is present at the motor shaft, equation 4.2 must be rewritten to read:

$$\ddot{\theta}_o + \frac{f}{J}\dot{\theta}_o + \frac{c}{J}\,\mathrm{sign}\,\dot{\theta}_o = K\varepsilon \qquad 4.3$$

Since $\frac{f}{J} = 1$ for the chosen system, and if $\frac{c}{J} \triangleq C'$, equation 4.3 can be rearranged to:

$$S\theta_o(s+1) = K\varepsilon - C'\,\mathrm{sign}\,\dot{\theta}_o$$

where initial condition was assumed to be zero.

$$S\theta_o = \frac{K\varepsilon - C'\,\mathrm{sign}\,\dot{\theta}}{S+1} \qquad 4.4$$

or

$$S\theta_o = \dot{\theta}_o$$

From equation 4.4, it is obvious that the transfer function method of simulation can be used for systems with coulomb friction if proper scaling is done. Then the block diagram can be established as shown in Fig. 4-1. If the coulomb friction is present at the load shaft instead of motor shaft, coulomb friction C can be referenced to the motor shaft by letting $C' \triangleq \frac{kg^2}{J}C$, where kg is gear ratio.

(b) Scaling.

Let the referenced coulomb friction torque C' be;

i $\quad C' = .06$

ii $\quad C' = .12$

Where it is assumed that the unit is in accord.

$$\bar{C'}\alpha C' = C'$$

where

$$\alpha C' \triangleq \frac{C'}{\bar{C'}}$$

One can choose any convenient value of $\bar{C'}$ for more convenient values of $\alpha C'$. Usually $\bar{C'}$ is chosen for a convenient value of limiting voltage of the coulomb simulation circuit. Batteries may well best be used in the limiter, because of the scaling convenience.

Going back to equation 4.4 in the problem form,

$$\dot{\theta}_0 = \frac{K\varepsilon - C' \operatorname{sign} \dot{\theta}_0}{S + 1}$$

This equation can be written in terms of the computer values as equation 4.5.

$$\bar{\dot{\theta}}\alpha\dot{\theta} = \left(\frac{1}{S + \frac{1}{\alpha t}}\right)\frac{1}{\alpha t}\left[K\bar{\varepsilon}\alpha\varepsilon - \bar{C'}\alpha C' \operatorname{sign} \dot{\theta}_0\right] \quad 4.5$$

where, in this example: $\alpha\varepsilon \triangleq \alpha\theta$

From equation 4.5, the scaling procedure is clear and self explanatory.

(c) Simulation circuit (Fig. 4-2)

where:

$r = 100K$

$R_1 \approx 2M.$

$R_2 = 1.00M$

$E = 100V$

Let: $\quad \alpha C' = .012$

$$c' \quad \text{i} \quad c' = .06 \qquad \bar{c}' = 5 = \frac{a_1 E}{1-a_1}$$

$$\text{ii} \quad c' = .12 \qquad \bar{c}' = 10 = \frac{a_1}{1-a_1} E$$

$$a_1 \quad \text{i} \quad a_1 = \frac{5}{E+5} = .0476$$

$$\text{ii} \quad a_1 = \frac{10}{E+10} = .0909$$

$$\left.\frac{a_1 r}{R_1}\right|_{max.} \cong .004$$

Rearranging equation 4.5,

$$\bar{\theta}_o = \frac{1}{\alpha\dot{\theta}\alpha t}\left[\bar{\varepsilon} K \alpha \theta - \bar{c}' \alpha c \operatorname{sign} \dot{\theta}\right]\left[\frac{1}{S + \frac{1}{\alpha t}}\right] \qquad 4.7$$

Let

$$\omega_c = \frac{\alpha c'}{\alpha\dot{\theta}\alpha t} = .1819$$

Also,

$$\frac{b}{R_2 C_f} = \omega_c \qquad b = .1891 \qquad R_2 = 1.00\ M. \qquad C_f = 1.04\ \mu f$$

(d) Resultant curves.

The effect of coulomb friction in the transient response of the system is as if the damping of system is increased. In Fig. 4-3 and 4-4, the overshoot becomes less when the coulomb friction is increased, while at the same time ouput velocity decreases.

Due to coulomb friction in the system, the output position will have a finite steady state error when step input is imposed. This steady state error will be; $\quad 0 \leq \varepsilon_{ss} \leq \frac{c'}{K}$

If

$$\bar{c}' = 5 \qquad \varepsilon_{ss} \leq 2.22\ V.$$

$$\bar{c}' = 10 \qquad \varepsilon_{ss} \leq 4.45\ V.$$

This effect is well shown in the phase plane plot of Fig. 4-5.

The frequency response curve shows a definite presence of nonlinearity in the system.

(e) Discussion.

In the case, $\frac{a_1 r}{R_4} \approx .004$, and the output of the limited quantity has a error of $\pm .004\ \dot{\theta}_o$.

This error could be reduced by the use of dry batteries in place of voltage sources and potentiometers. Also the same objective can be accomplished if lower resistance potentiometer or higher voltage sources were used.

Frequency response data
(coulomb friction)

$\bar{\tau}' = 5$

$\bar{f}$	ω	$\bar{\Theta}_o$	M	$\bar{\Theta}_o$	M
.01	.0943	19.2	.96	39.5	.988
.02	.1885	20.2	1.01	41	1.025
.03	.283	21.2	1.06	44	1.1
.04	.377	23.7	1.185	48	1.2
.05	.472	26	1.3	54	1.35
.06	.566	28	1.4	59	1.475
.065	.613	28.5	1.425	60	1.5
.07	.66	28	1.4	59	1.475
.08	.754	25.8	1.29	55	1.375
.09	.848	21.2	1.06	45	1.125
.1	.943	16.5	.825	35.5	.888
.2	1.885	3	.15	6.5	.1625
.4	3.77	.7	.035	1.4	.035
.6	5.66	.26	.013	.6	.015
.8	7.54	.14	.007	.3	.0075
1.0	9.43	.05	.0025	.14	.0035

$\bar{\Theta}_i = 20 \sin \omega t$ $\quad\quad$ $\bar{\Theta}_i = 40 \sin \omega t$

Frequency response data
(coulomb friction)
$\bar{C}' = 10$ T.

$\bar{f}$	ω	$\bar{\Theta}_o$	M	$\bar{\Theta}_o$	M
.01	.0943	18	.9	36.7	.918
.02	.1885	18.8	.94	39	.975
.03	.283	20	1.0	42.2	1.055
.04	.377	22.5	1.125	47.5	1.19
.05	.472	24	1.2	52.5	1.312
.06	.566	25.9	1.295	57	1.425
.065	.613	26	1.3	57.5	1.44
.07	.66	25.8	1.29	57	1.425
.08	.754	23	1.15	52.5	1.312
.09	.848	19.5	.975	43	1.075
.1	.943	15.9	.795	34	.84
.2	1.885	3	.15	6.6	.165
.4	3.77	.7	.035	1.5	.0375
.6	5.66	.25	.0125	.6	.015
.8	7.54	.14	.007	.24	.006
1.0	9.43	.05	.0025	.15	.00375

$\bar{\Theta}_i = 20 \sin \omega t$ $\qquad$ $\bar{\Theta}_i = 40 \sin \omega t$

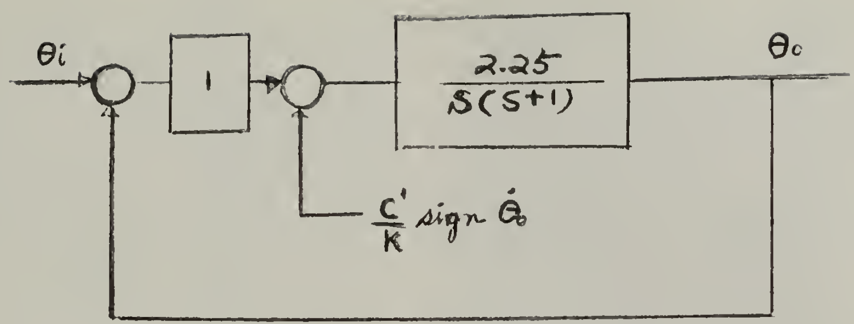


Fig. 4-1 Block diagram of the system with Coulomb friction.

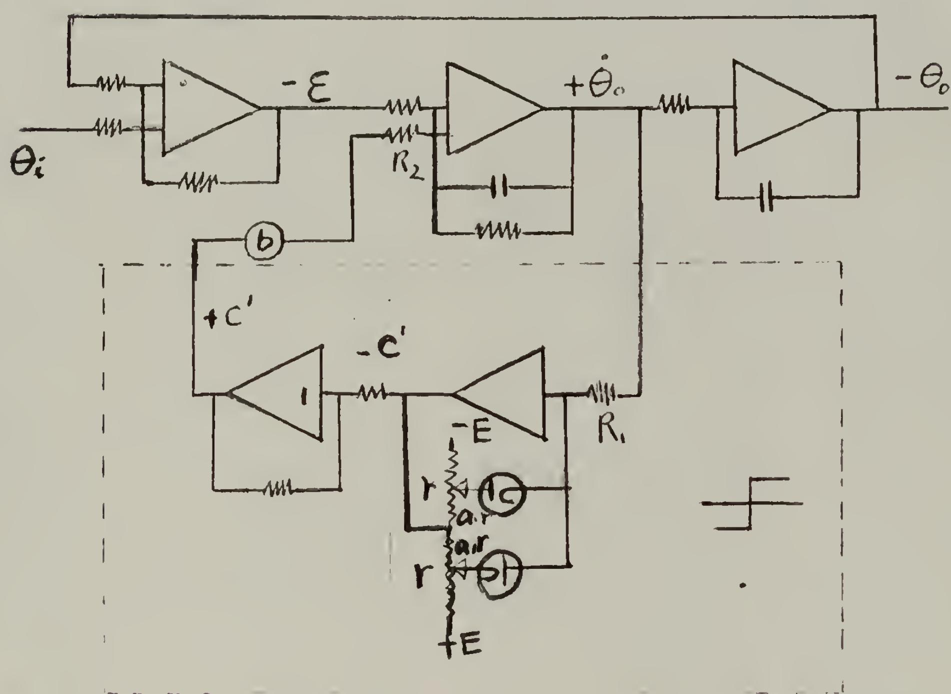


Fig. 4-2 Simulation circuit for the system shown in Fig. 4-1.

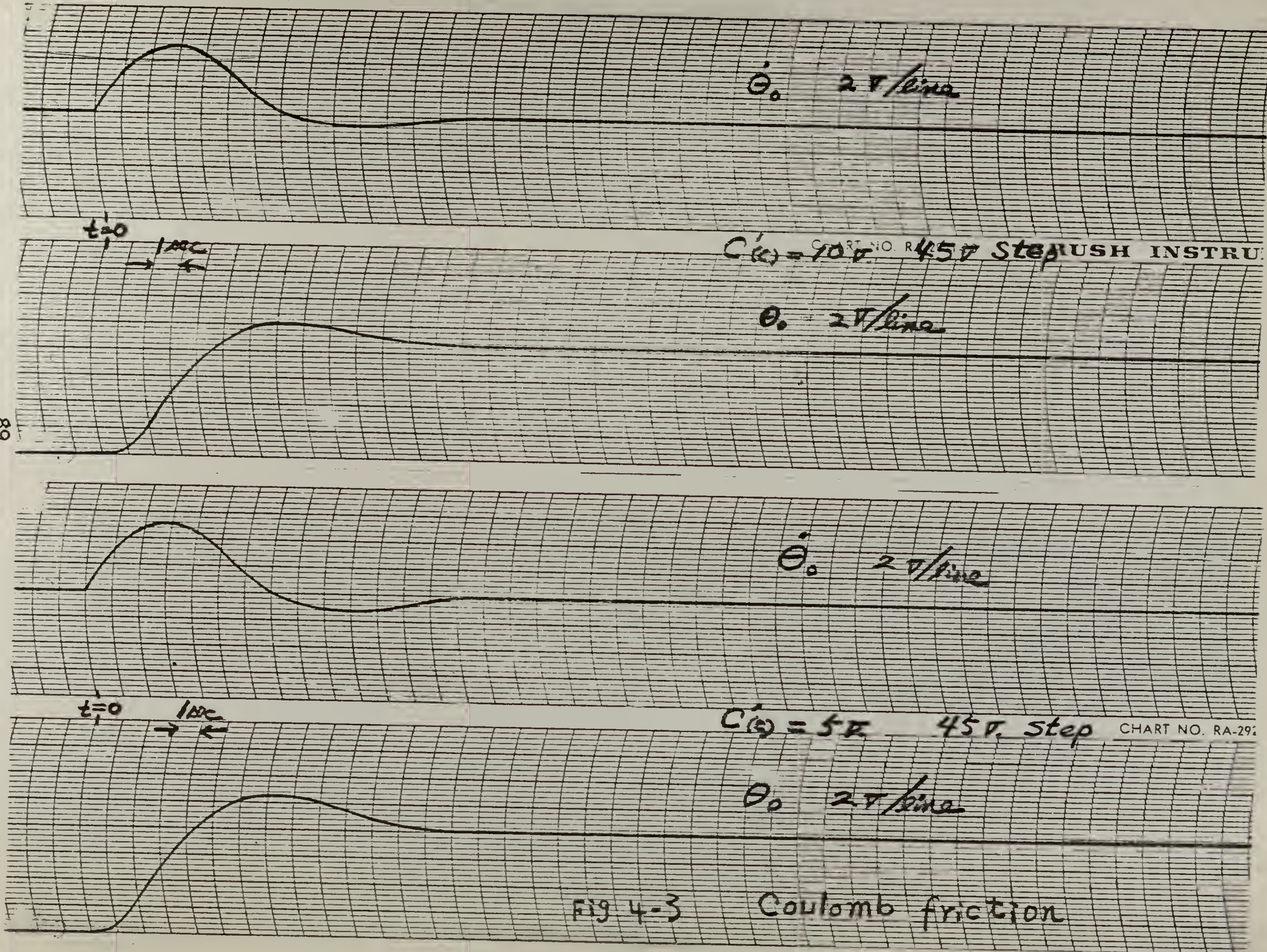
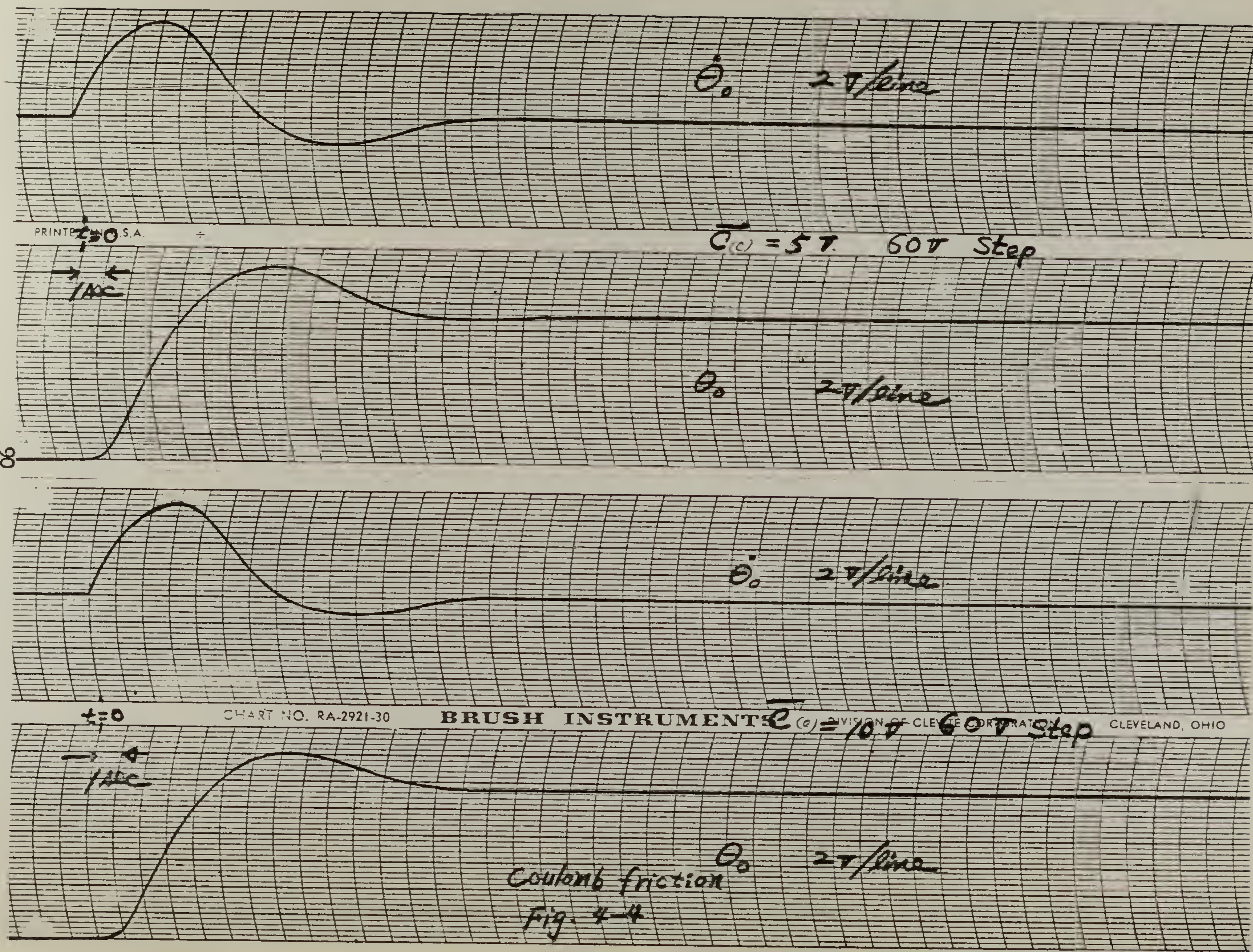
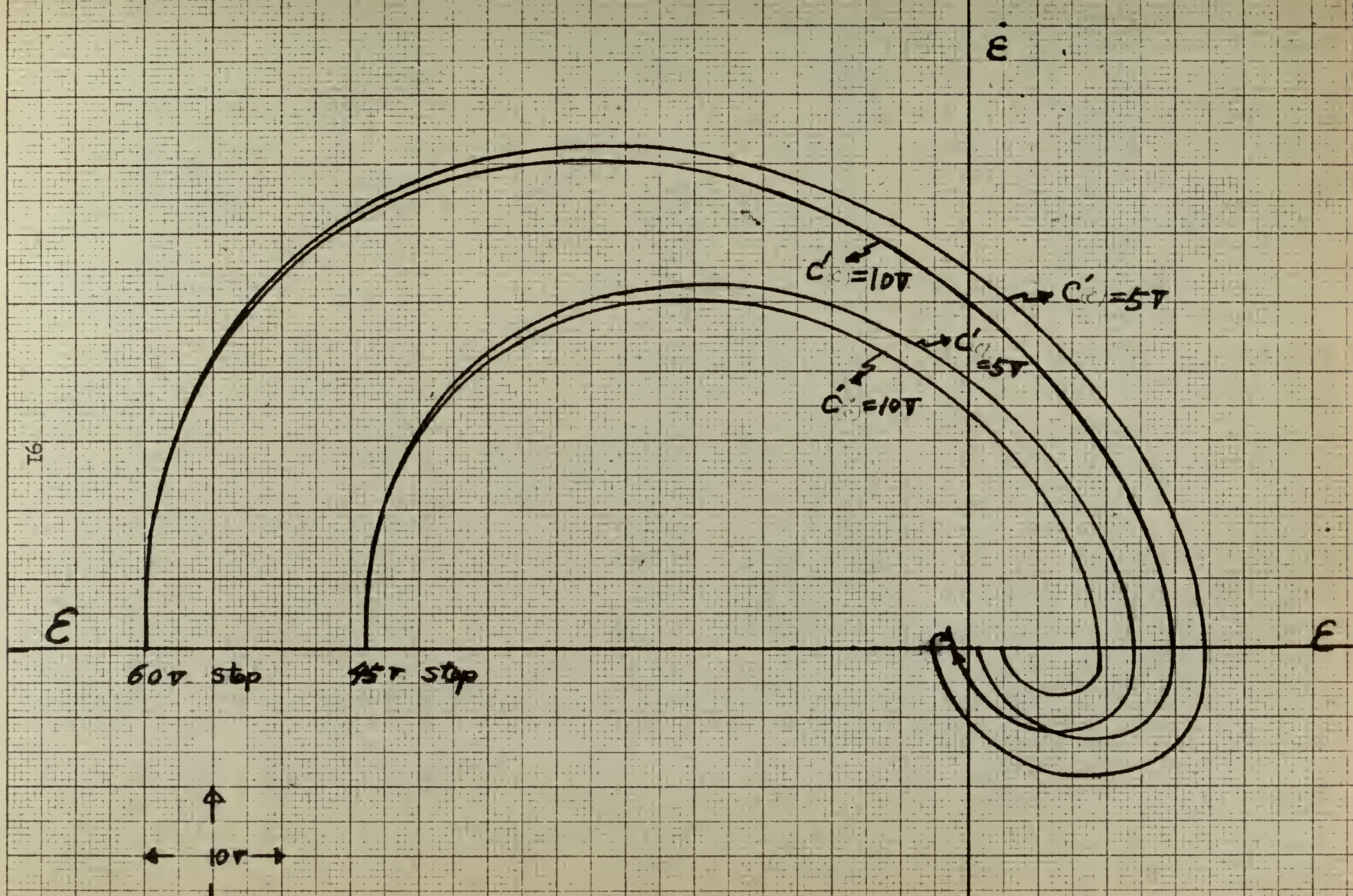


Fig 4-3 Coulomb friction

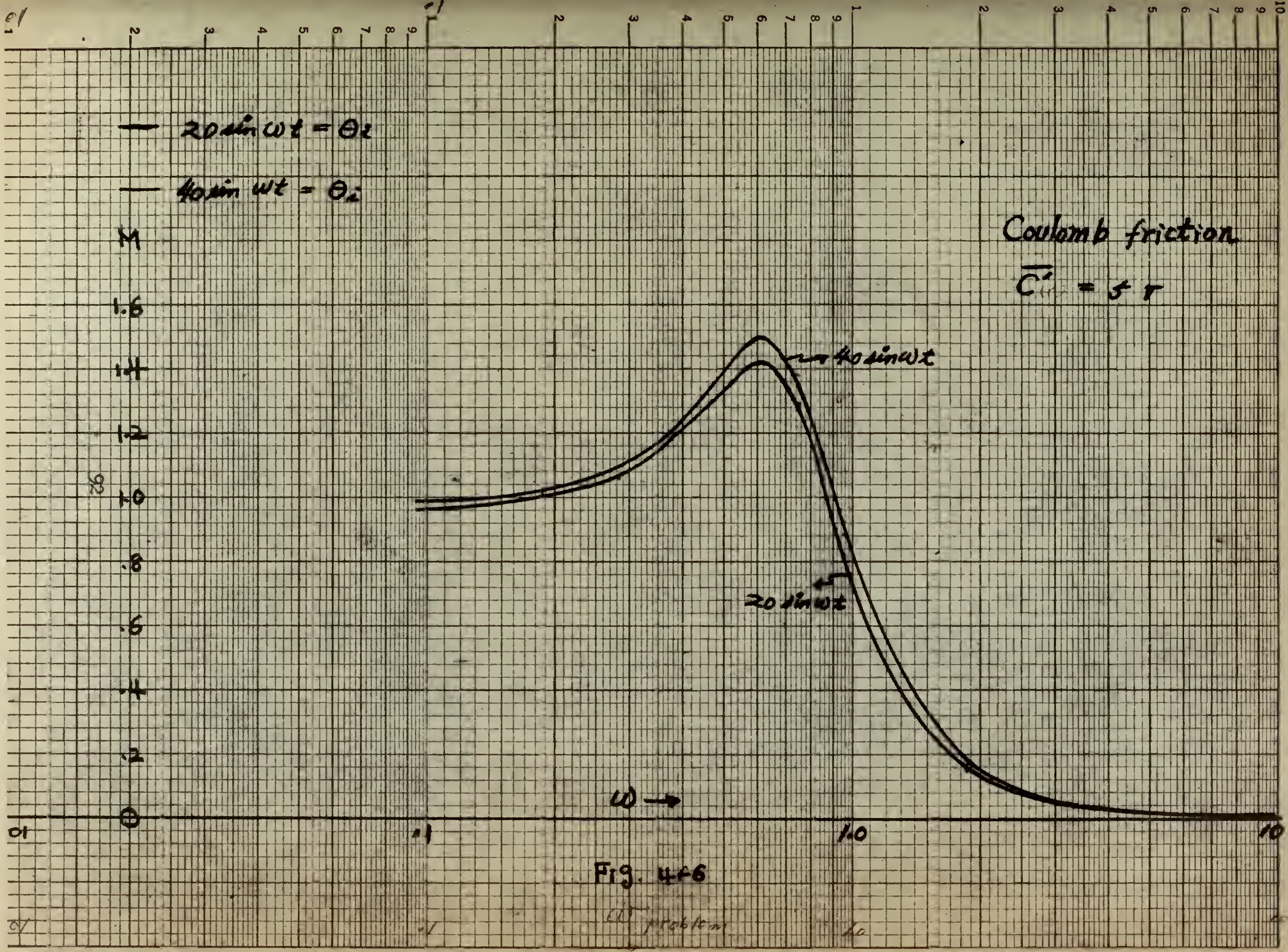


Coulomb friction
Fig. 4-4



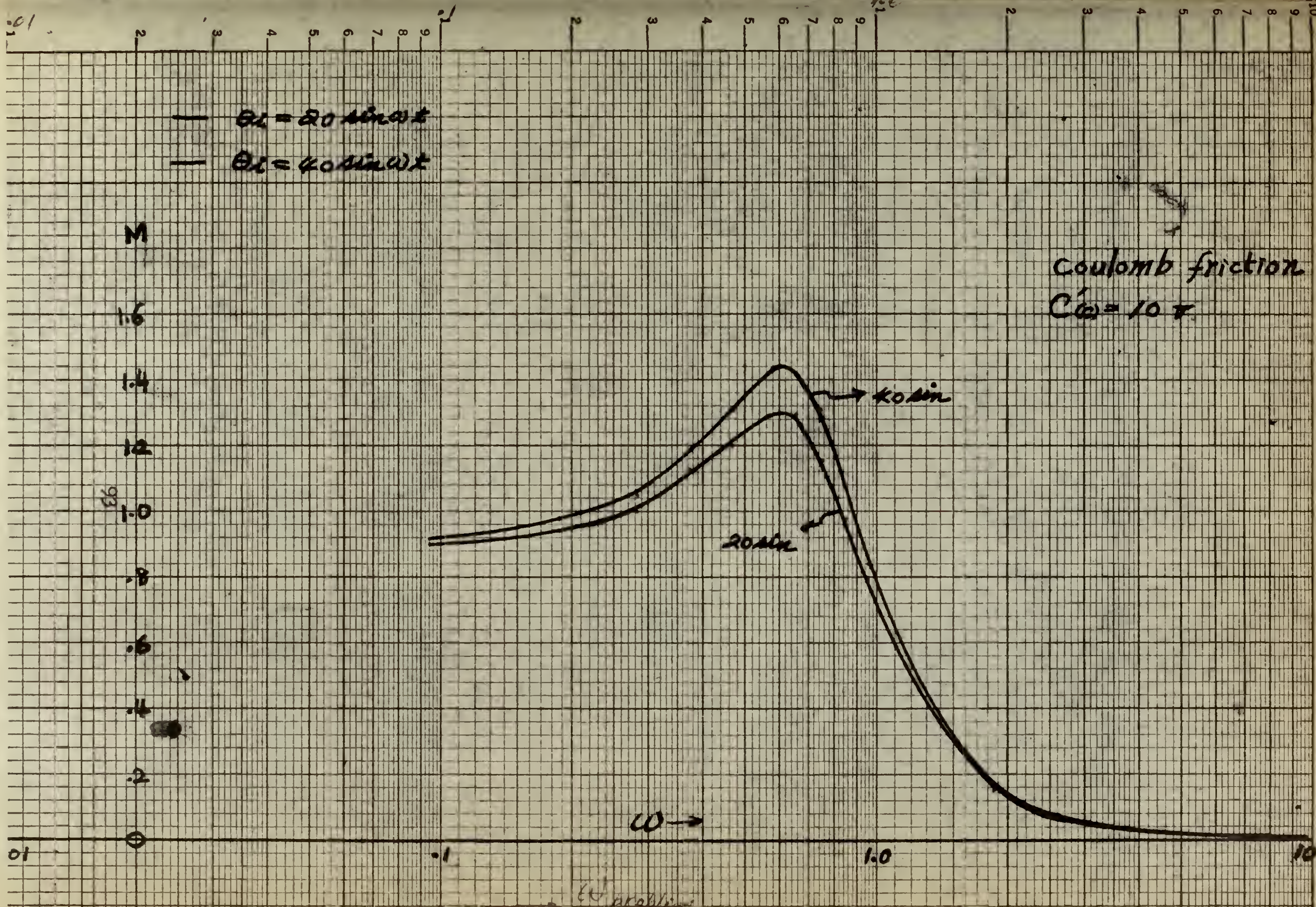
Coulomb friction

Fig. 4-5



$20 \sin \omega t = \theta_i$
$40 \sin \omega t = \theta_i$
M
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
Coulomb friction
$\overline{C'} = 5\,r$
40 sin ωt
20 sin ωt
ω →
.1
1.0
10

Fig. 4-6



$\theta_i = 20 \sin\omega t$
$\theta_i = 40 \sin\omega t$
M
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
Coulomb friction
$C'_{(\omega)} = 10$ #
40 sin
20 sin
$\omega \rightarrow$
.01
.1
1.0
10

Fig 4-7

E. Simple backlash.

(a) Block diagram (Fig. 5-1)

Backlash is assumed to be present in the load shaft of the system, and the output position is measured at the load shaft.

The simple backlash implies that the load inertia is negligible compared to the drag force so that the load does not drift during backlash period and when gears recombine, load shaft velocity jumps to that of the motor.

(b) Scaling.

The scaling of this problem is really simple. Since the quantity to be dealt is the position, and the circuit is to be inserted to the main branch, the backlash Δ must be scaled to the computer value by applying positional scaling factor. $\bar{\Delta} = \dfrac{\Delta}{\alpha\theta_0}$

Let;

i $\Delta = .05$ $\qquad \bar{\Delta} = \dfrac{.05}{.015} = 3.33$

ii $\Delta = .1$ $\qquad \bar{\Delta} = \dfrac{.1}{.015} = 6.667$

(c) Simulation circuit (Fig. 5-2).

Where;

$E = 45$

$R = .1$ M.

$C = 1\,\mu f$

a_1 i $\dfrac{\bar{\Delta}}{2} = \dfrac{a_1}{1-a_1} E = \dfrac{3.33}{2}$ $\qquad a_1 = .0357$

ii $\dfrac{\bar{\Delta}}{2} = \dfrac{a_1}{1-a_1} E = \dfrac{6.667}{2}$ $\qquad a_1 = .0688$

From paragraph D of Section 2,

$$\theta_o = \frac{(1+RCS)(-\theta_1 + \frac{a_1}{1-a_1}E)}{1 + \delta RCS} \qquad \delta \triangleq 1 + \frac{a_1}{1-a_1} + \frac{a_1 r}{R_1} + \frac{a_1 r}{R_3}$$

Since potentiometer loading effect due to R1 and R3 are taken care of;

$$\delta \approx \frac{1}{1-a_1} = 1.04 \qquad \text{for} \quad a_1 = .0357$$

$$= 1.075 \qquad \text{for} \quad a_1 = .0688$$

$$\theta_o \cong \theta_1 \pm \frac{a_1}{1-a_1} E$$

$$R_2 = .025\ M \qquad C_2 = 1.034\ \mu f \qquad R_f = 5.02 M$$

$$\mathring{\theta} \approx b R_f C_2 S \dot{\theta}_o$$

$$b = \frac{1}{R_f C_2} \frac{\alpha + \alpha\theta}{\alpha \dot{\theta}} = .395$$

(d) Resultant curves.

It is a well established fact that the backlash has a destabilizing effect on the system performance. From Fig. 4-3 and 4-4 of transient response curves, it is seen that the overshoot and output velocity becomes greater with more oscillations when the amount of backlash is increased.

Fig. 5-5 and 5-6 of phase plane plots, show more clearly of this situation. For the system damping factor of .333, of second order system, there should be no limit cycle, whereas the simulation shows a limit cycle for reasons that are not quite clear.

However, with the understanding of the relation between maximum damping factor that will give limit cycle in the system, this circuit can be utilized for general engineering purposes.

Frequency response plot on Fig. 5-7 and 5-8, shows that the increase of backlash corresponds to a greater $M_{p\omega}$. $M_{p\omega}$ is generally

higher than that of the linear system which is shown in Fig. 1-5.

For larger input signal, backlash plays less effect on the system, thus becoming close to linear system performance, which in turn will indicate a decrease in M_{pw}. These effects are well shown on the frequency response curves.

(e) Discussion.

i. If E in the Fig. 5-2 is increased to higher value, and if the potentiometer loading has been corrected, δ can become closer to the unity.

ii. As resultant response curves indicate, the sumulation shows limit cycle contradicting to the behavior of the physical system. Although the reasons are not quite clear, the writer suspects it to be caused from stored energy in the capacitor C of Fig. 5-2.

Frequency response data
(simple backlash)

$\bar{\Delta} = 3.33$

		$\bar{\theta}_i = 20 \sin \omega t$		$\bar{\theta}_i = 40 \sin \omega t$	
$\bar{f}$	ω	$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	21.5	1.075	42	1.05
.02	.1885	22	1.1	43.7	1.09
.03	.283	23.8	1.19	48	1.2
.04	.377	26	1.3	53	1.325
.05	.472	29	1.45	58.5	1.46
.06	.566	32.7	1.635	64.5	1.612
.068	.642			68	1.7
.07	.66	34.2	1.71	67.5	1.687
.071	.668	34.3	1.715	67.5	1.687
.072	.679	34	1.7	67.5	1.687
.075	.707	33.5	1.625	66.5	1.66
.08	.754	31	1.55	62.5	1.56
.09	.848	25.5	1.275	51	1.275
.1	.943	19.5	.975	38	.95
.2	1.885	2	.1	5	.125
.4	3.77	.5	.025	.2	.005
.6	5.66	.2	.01	.2	.005
1.0	9.43	.15	.0075	.15	.00375

Frequency response data
(simple backlash)

$\bar{\Delta} = 6.667$

$\bar{f}$	ω	$\bar{\theta}_i = 20 \sin \omega t$ $\bar{\theta}_o$	M	$\theta_i = 40 \sin \omega t$ $\bar{\theta}_o$	M
.01	.0943	21	1.05	40.8	1.02
.02	.1885	21.5	1.075	43	1.075
.03	.283	22.5	1.125	44.5	1.11
.04	.377	25.5	1.275	49	1.225
.05	.472	30	1.5	57	1.425
.06	.566	34.7	1.735	64.5	1.61
.068	.642	36.7	1.835	67.5	1.688
.07	.66	36.7	1.835	67	1.675
.072	.679	36.4	1.82	67	1.675
.074	.698	35.7	1.785	66	1.65
.08	.754	32	1.6		
.09	.848	24	1.2	48	1.2
.1	.943	17	.85	37.6	.938
.2	1.885	4	.2	3	.075
.4	3.77	1.2	.06	1.5	.0375
.6	5.66	.6	.03	.9	.0225
.8	7.54	.3	.015	.5	.0125
1.0	9.43	.2	.01	.3	.0075

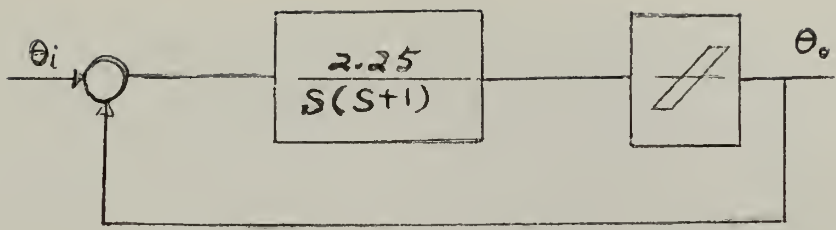


Fig. 5-1 Block diagram of the system with simple backlash.

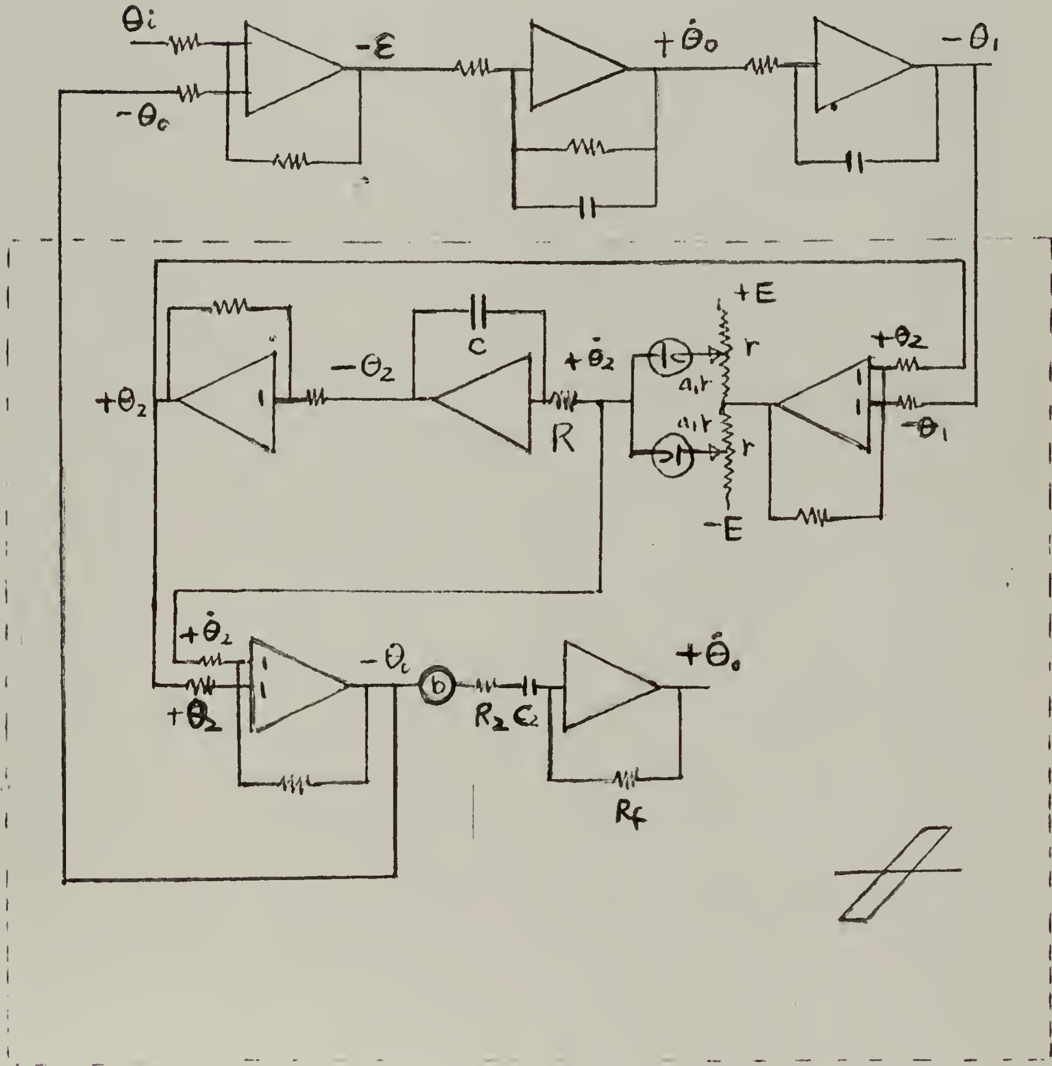
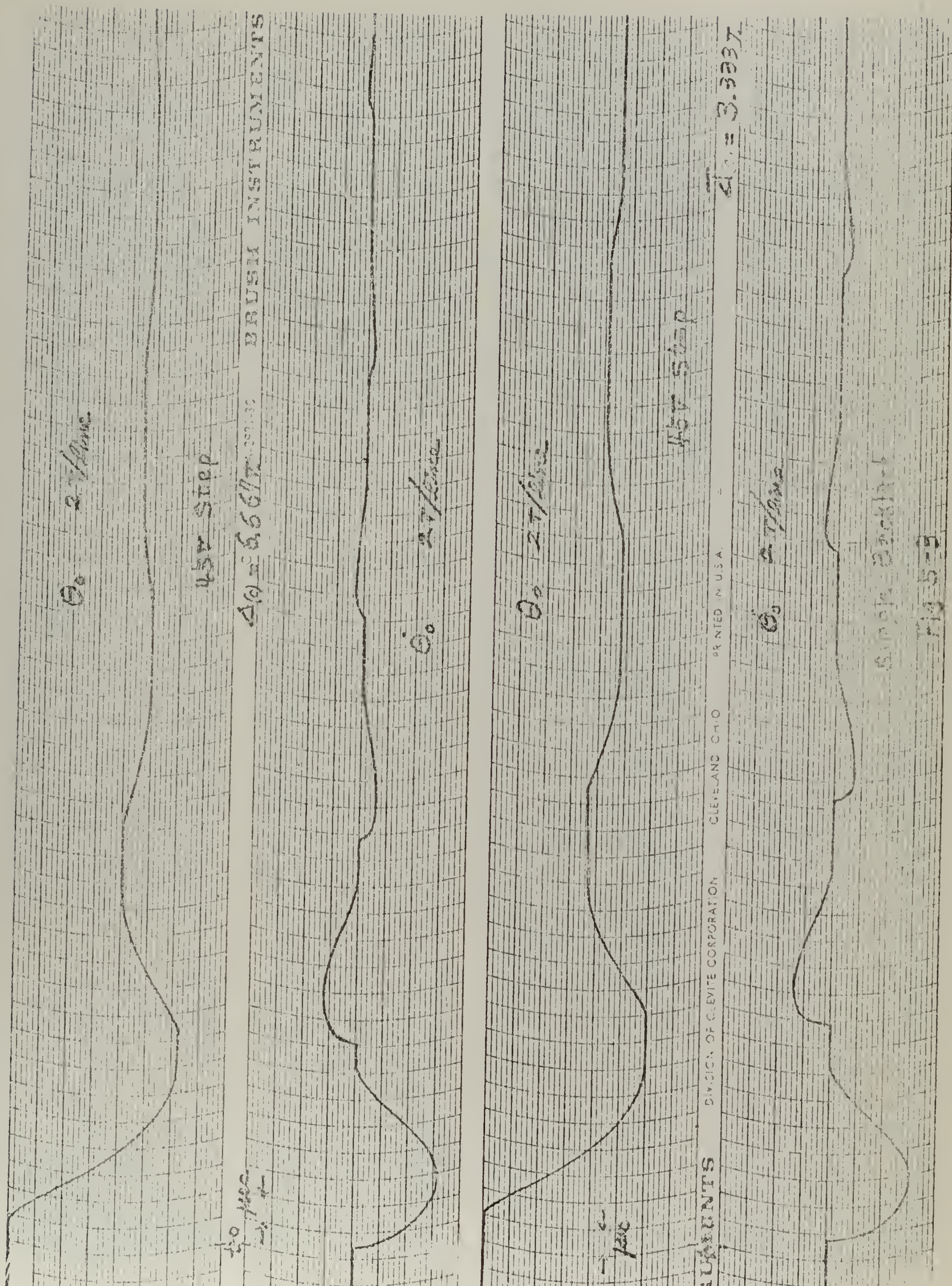
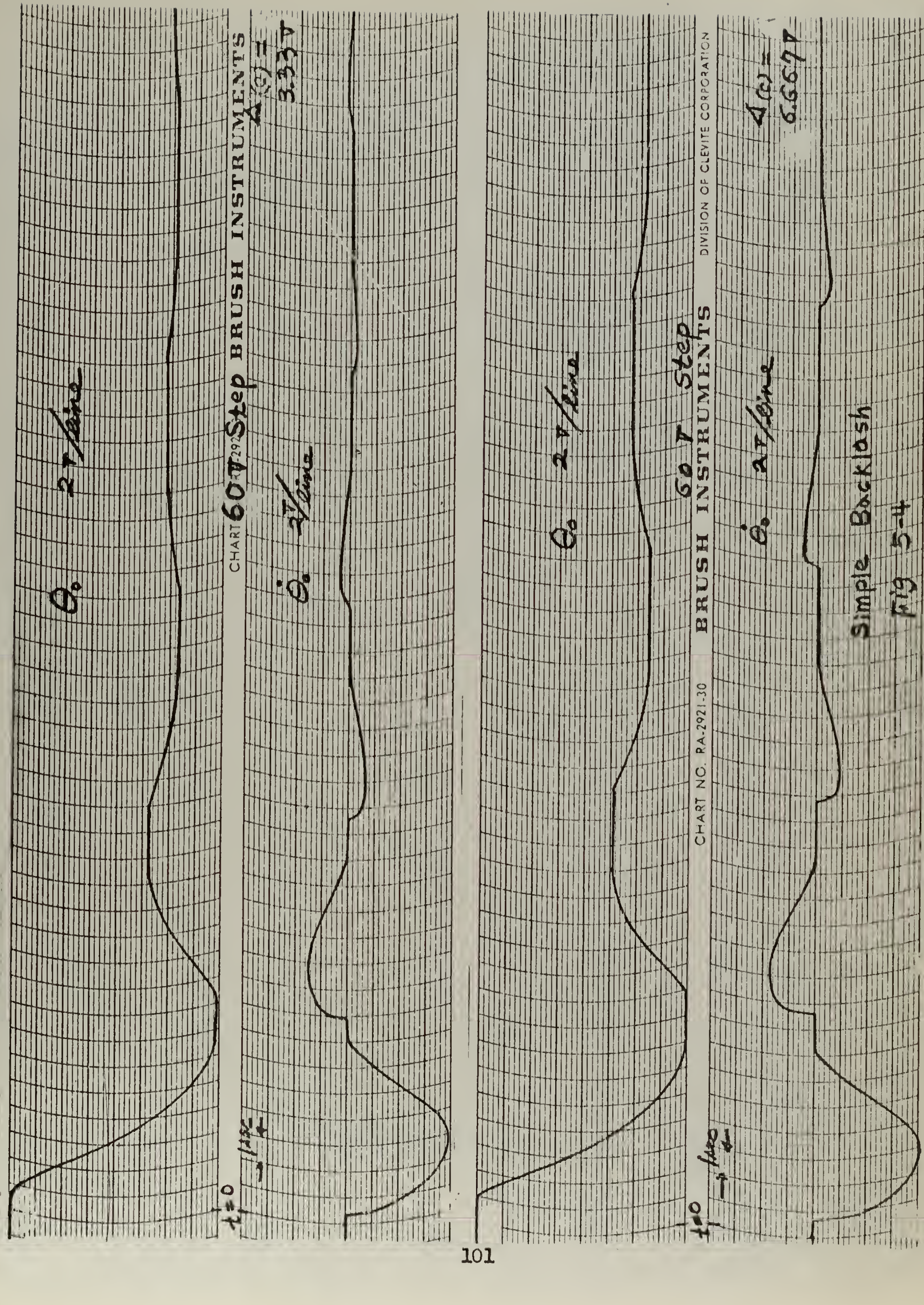
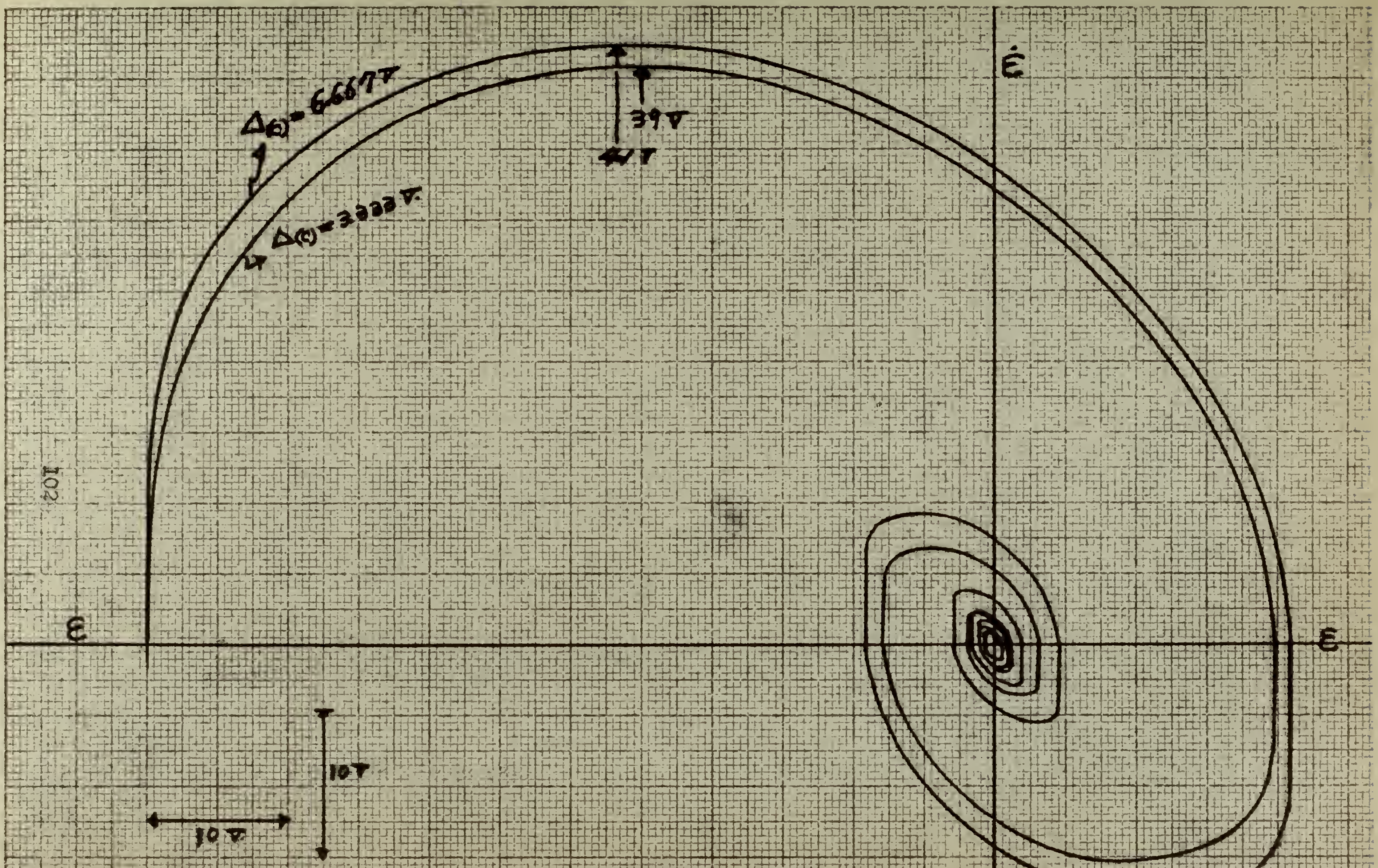


Fig.5-2 Simulation circuit for the system shown in Fig. 5-1.


BRUSH INSTRUMENTS
45° Step
DIVISION OF CLEVITE CORPORATION
CLEVELAND OHIO
PRINTED IN U.S.A.
45° Step

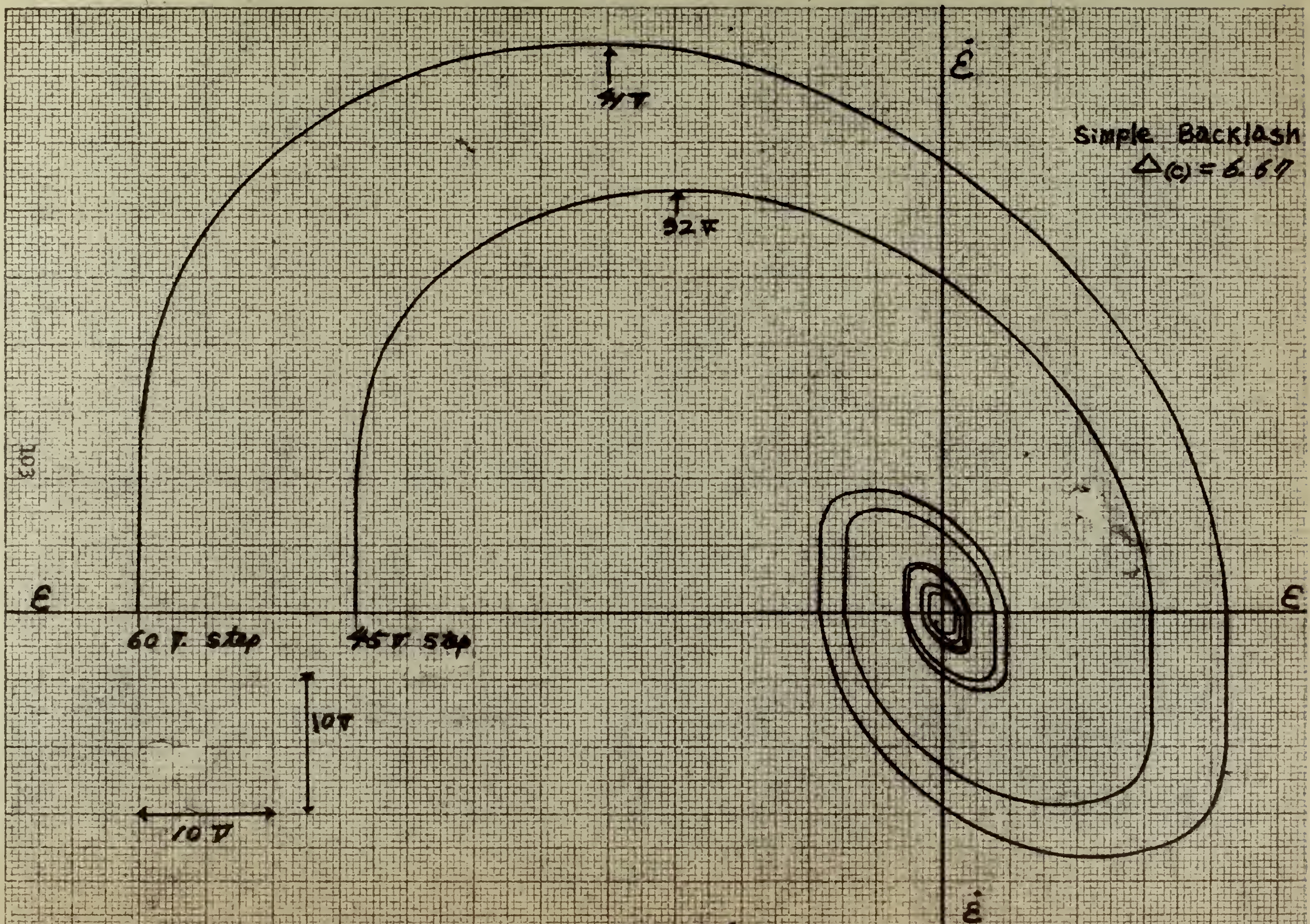


Simple Backlash
Fig 5-4



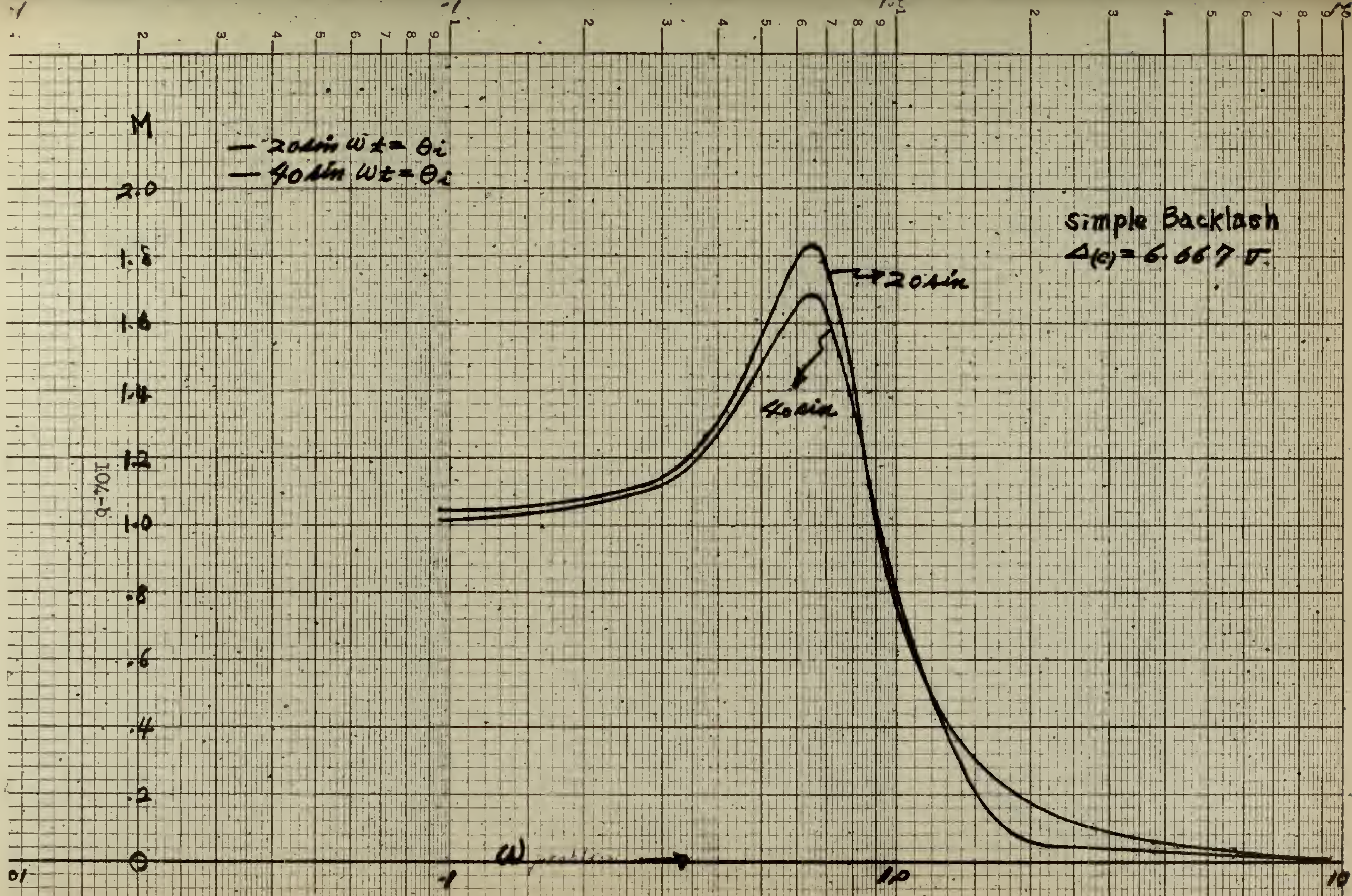
Simple Backlash
60 V step

Fig. 5-5



$\dot{\mathcal{E}}$
Simple Backlash
$\Delta(c) = 6.6V$
32 V
$\mathcal{E}$
60 V. step
45 V step
10 V
10 V
$\mathcal{E}$
$\dot{\mathcal{E}}$

Fig. 5-6



M
— 20 sin ωt = θi
— 40 sin ωt = θi
Simple Backlash
Δ(c) = 6.667 v
20 sin
40 sin
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
.01
.1
1.0
10
ω

Fig 5-8

104-b

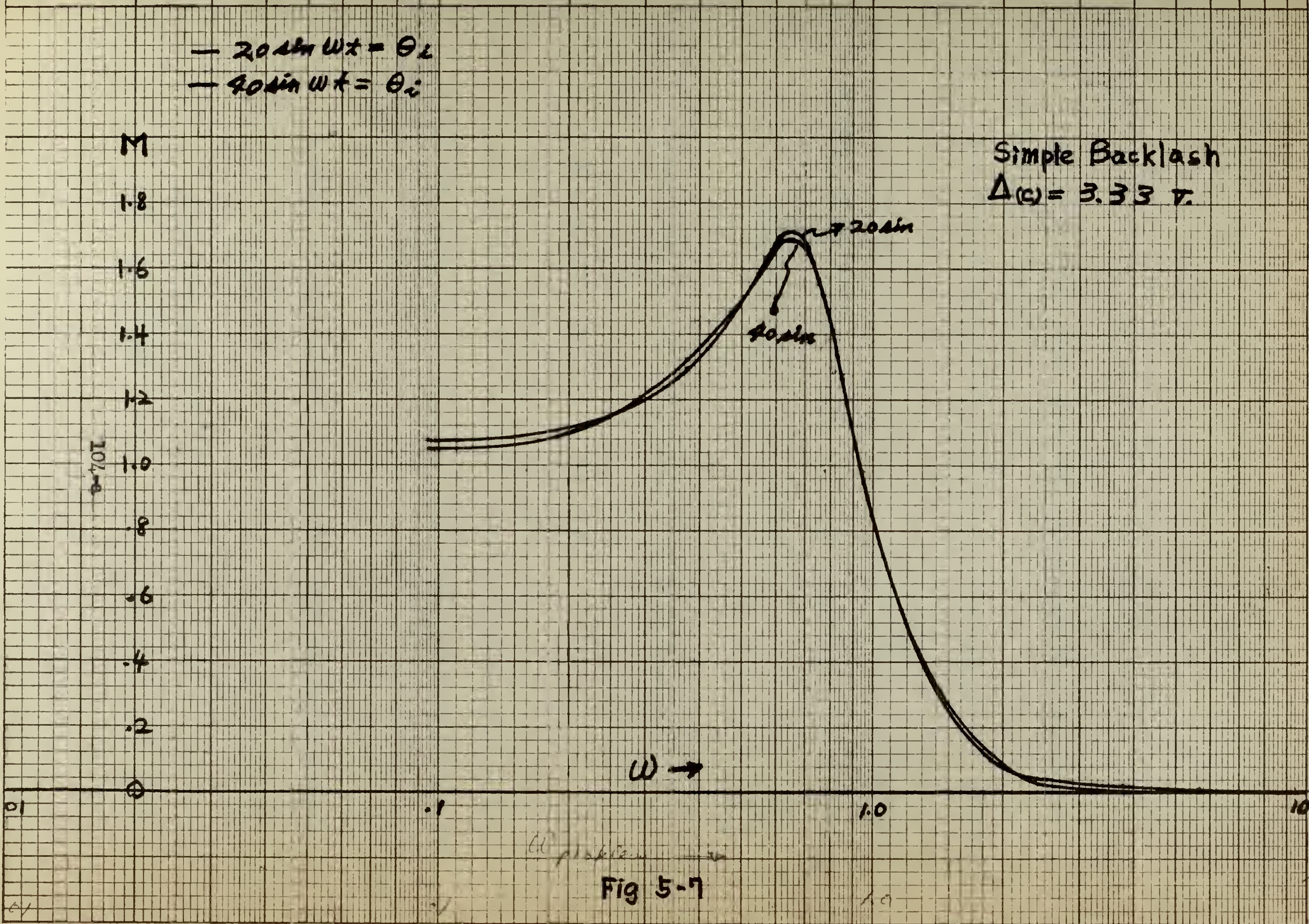

— 20 sin ωt = θi
— 40 sin ωt = θi
M
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
Simple Backlash
Δ(c) = 3.33 v.
20 sin
40 sin
ω →
.1
1.0
10

Fig 5-7

F. Ideal relay.

(a) Block diagram.

The ideal relay is placed in the error channel as shown in Fig. 6-1.

In all previous cases, motor gain was assumed to be included in the constant $K = 2.25$.

But, for simulations of a relay servo, it is assumed that $K = 2.25$ <u>does not</u> include the motor gain constant, and it describes the open loop gain of the system not including motor. Also the motor is assumed to be an ideal torque motor.

The equations describing the system become:

$$J\ddot{\theta}_0 + f\dot{\theta}_0 = +T \qquad \varepsilon > 0 \qquad 6.1$$

$$J\ddot{\theta}_0 + f\dot{\theta}_0 = -T \qquad \varepsilon < 0 \qquad 6.2$$

where T denotes the constant torque output of the motor.

If $f = J = \frac{1}{2.25}$

$$\ddot{\theta} + \dot{\theta} = 2.25(T) \qquad \varepsilon > 0$$

$$\ddot{\theta} + \dot{\theta} = 2.25(-T) \qquad \varepsilon < 0$$

Then the transfer function set up used for simulation of all cases in previous paragraphs can be used without changing scaling factors and circuit parameters.

(b) Scaling.

Assuming that the torque T is in proper units the system equation can be arranged as follows:

i. $\varepsilon > 0$

$$S\theta_0(1+S) = 2.25\,T$$

$$\dot{\theta}_0 = \frac{2.25\,T}{1+S}$$

ii. $\varepsilon < 0$

$$\mathring{\Theta}_o = \frac{2.25}{1 + S}(-T)$$

And the computer equations are;

$$\bar{\mathring{\theta}}_o \alpha \mathring{\theta} = \frac{2.25}{S + \frac{1}{\alpha t}} \left(\frac{\alpha T}{\alpha t}\right) \bar{T} \qquad \varepsilon > 0 \qquad 6.3$$

$$\bar{\mathring{\Theta}} \alpha \mathring{\theta} = \frac{2.25}{S + \frac{1}{\alpha t}} \left(\frac{\alpha T}{\alpha t}\right)(-\bar{T}) \qquad \varepsilon < 0 \qquad 6.4$$

From these equations, it is seen that the scaling is very simple, similar to the coulomb friction case.

If

$$\alpha T = \alpha \theta = .015,$$

linear system simulation circuit can be used without rescaling.

Let

$$T \quad \begin{array}{lll} \text{i} & T = .45 & \bar{T} = 30\ \text{V.} \\ \text{ii} & T = .75 & \bar{T} = 50\ \text{V.} \end{array}$$

(c) Simulation circuit (Fig. 6-2)

where:

$R_1 = 2.12$ M.

$r = 100$ K

$E = 200$ V.

$$a_1 \quad \begin{array}{lll} \text{i} & \frac{a_1}{1-a_1} E = \bar{T} = 30 & a_1 = .1304 \\ \text{ii} & \frac{a_1}{1-a_1} E = \bar{T} = 50 & a_1 = .2 \end{array}$$

$$\left.\frac{a_1 r}{R_1}\right|_{max.} \approx .01$$

(d) Resultant curves.

Theoretically, the system must be zeroed in to the origin in the

phase plane. The simulated results shows limiting cycle contradictory to the theory. This is due to noise effect in the amplifier since the loop gain is infinity.

In terms of transient response, the stability is poor. The stability becomes poorer if the motor torque is increased, exhibiting higher overshoot and output velocity. These effects are shown in both transient response curves and phase plane plots.

The frequency response curves are ploted as shown in Fig. 6-8 and 6-9. The portion up to the resonance peak in the curve, is not a true response, because the output was an oscillatory wave. The nonlinearity is well verified to be present in the system by noting different responses of the system to different sizes of input.

(e) Discussion.

Although the second order system with damping chosen must not exhibit limit cycle theoretically, the physical system will have noise involved, which in turn will drive the motor even when the error signal is zero. Considering this fact, the limit cycle observed in the simulated circuit is not a detrimental factor. If a resistor is shunted across the feed back path, it eventually zeroes in to the origin, but not without a finite time sacrifice for the output to get up to relay output voltage. The writer preferred the infinite gain open loop circuit.

Frequency response data
(ideal relay)

$\bar{T} = 30$

$\bar{f}$	ω	$\bar{\Theta}_o$	M	$\bar{\Theta}_o$	M
.01	.0943	20	1	40	1
.03	.283	20.1	1.005	40	1
.04	.377	20.3	1.015	40.1	1.002
.05	.472	20.3	1.015	40.5	1.015
.06	.566	20.5	1.025	41	1.025
.061	.575			49.5	1.238
.062	.584			48	1.2
.064	.603			45	1.125
.07	.66	20.5	1.025	39.5	.988
.074	.698	21	1.05		
.076	.716	33	1.65		
.078	.735	32	1.6		
.08	.754	31	1.55	31.5	.787
.09	.848	25	1.25	25.5	.613
.1	.943	20.5	1.025	21	.525
.2	1.885	5.1	.26	5.3	.1325
.4	3.77	1.3	.0715	1.33	.0323
.6	5.66	.6	.03	.5	.0125
1.0	9.43	.2	.01	.17	.00425
		$\bar{\Theta}_i = 20 \sin \omega t$		$\bar{\Theta}_i = 40 \sin \omega t$	

Frequency response data
(ideal relay)
$\bar{T} = 50$

$\bar{f}$	ω	$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	20	1	40.2	1.005
.03	.283	20.2	1.01	40.5	1.013
.04	.377	20.2	1.01	40.7	1.019
.05	.472	20.6	1.03	40.8	1.021
.06	.566	20.7	1.035	41	1.025
.07	.66	20.7	1.035	41.6	1.04
.08	.754	21	1.05	42	1.05
.081	.763			48	1.2
.086	.811			45	1.125
.09	.848	21	1.05	42.8	1.071
.1	.943	21	1.05	36	.9
.115	1.084	25	1.25		
.12	1.132	27.4	1.37		
.13	1.225	20.5	1.025		
.2	1.885	8.5	.425	8.7	.217
.4	3.77	2.2	.11	2.3	.575
.6	5.66	1.0	.05	1.0	.025
1.0	9.43	.3	.015	.28	.007

$\bar{\theta}_i = 20 \sin \omega t$ $\bar{\theta}_i = 40 \sin \omega t$

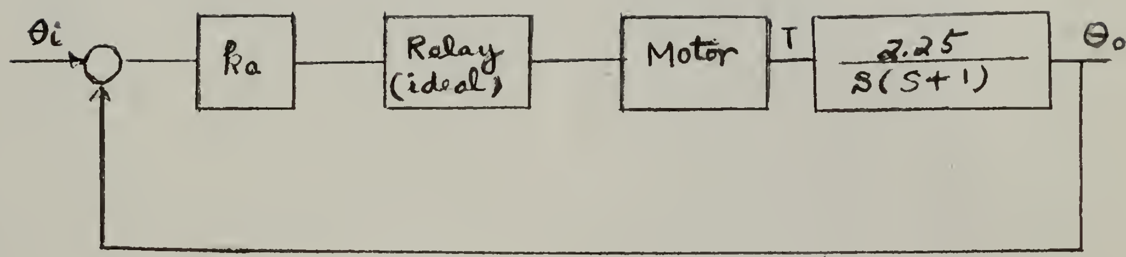


Fig. 6-1 Block diagram of the system with ideal relay.

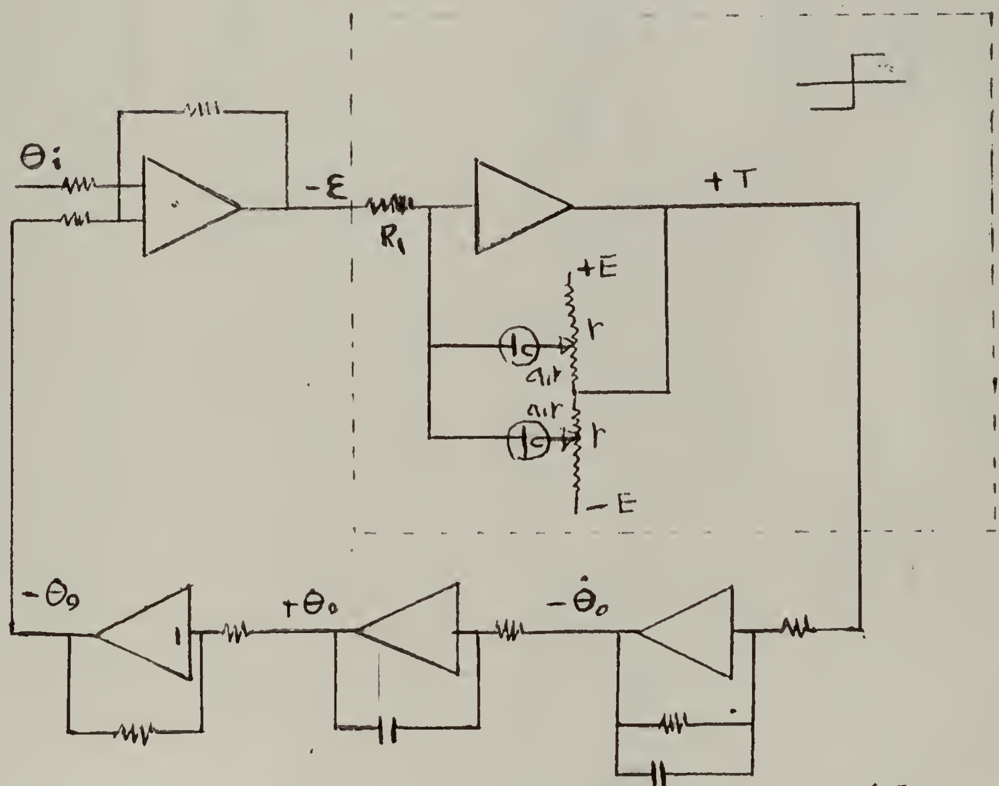
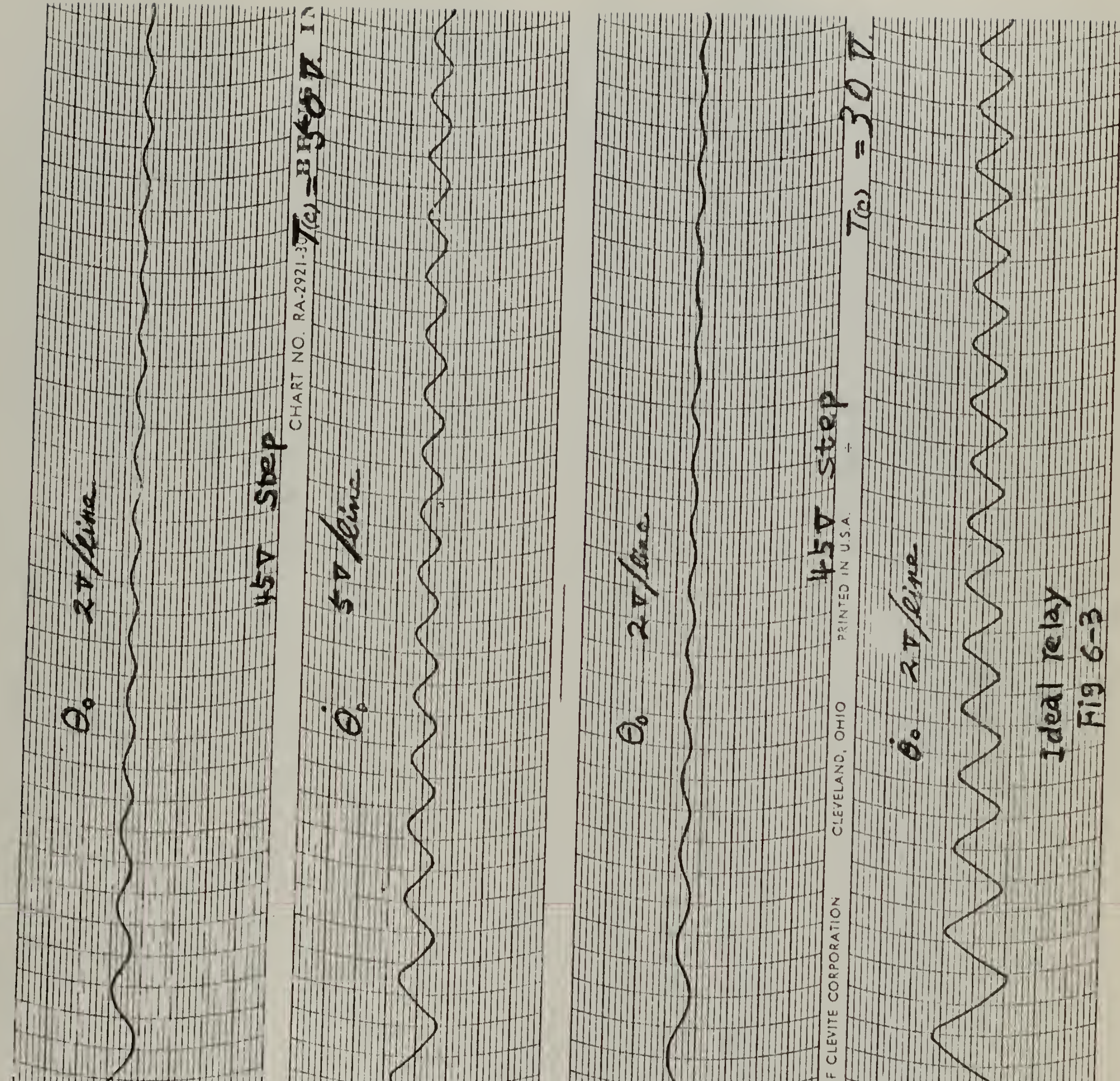
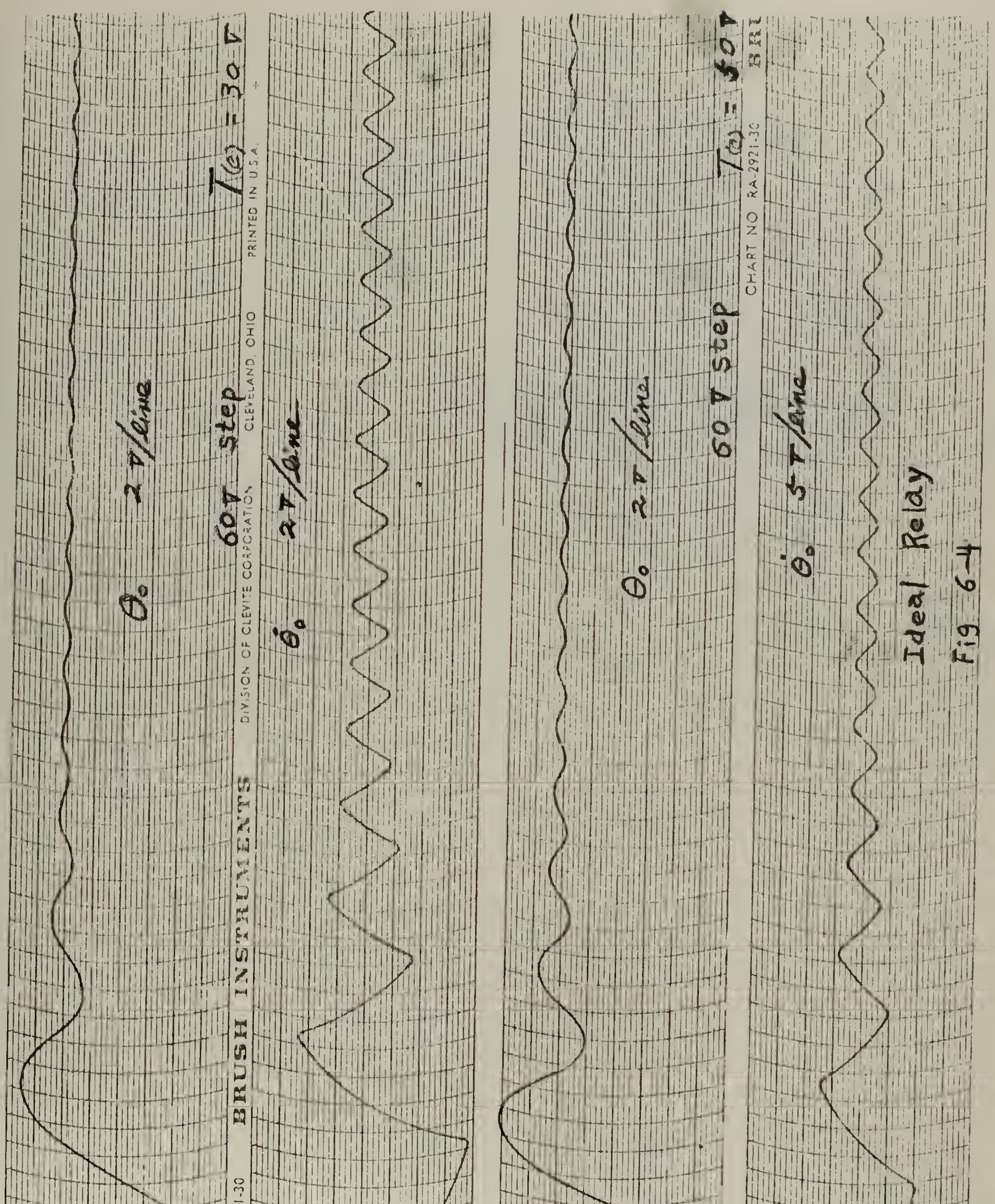


Fig.6-2 Simulation circuit for the system shown in Fig. 6-1.



Ideal relay
Fig 6-3



Ideal Relay

Fig 6-4

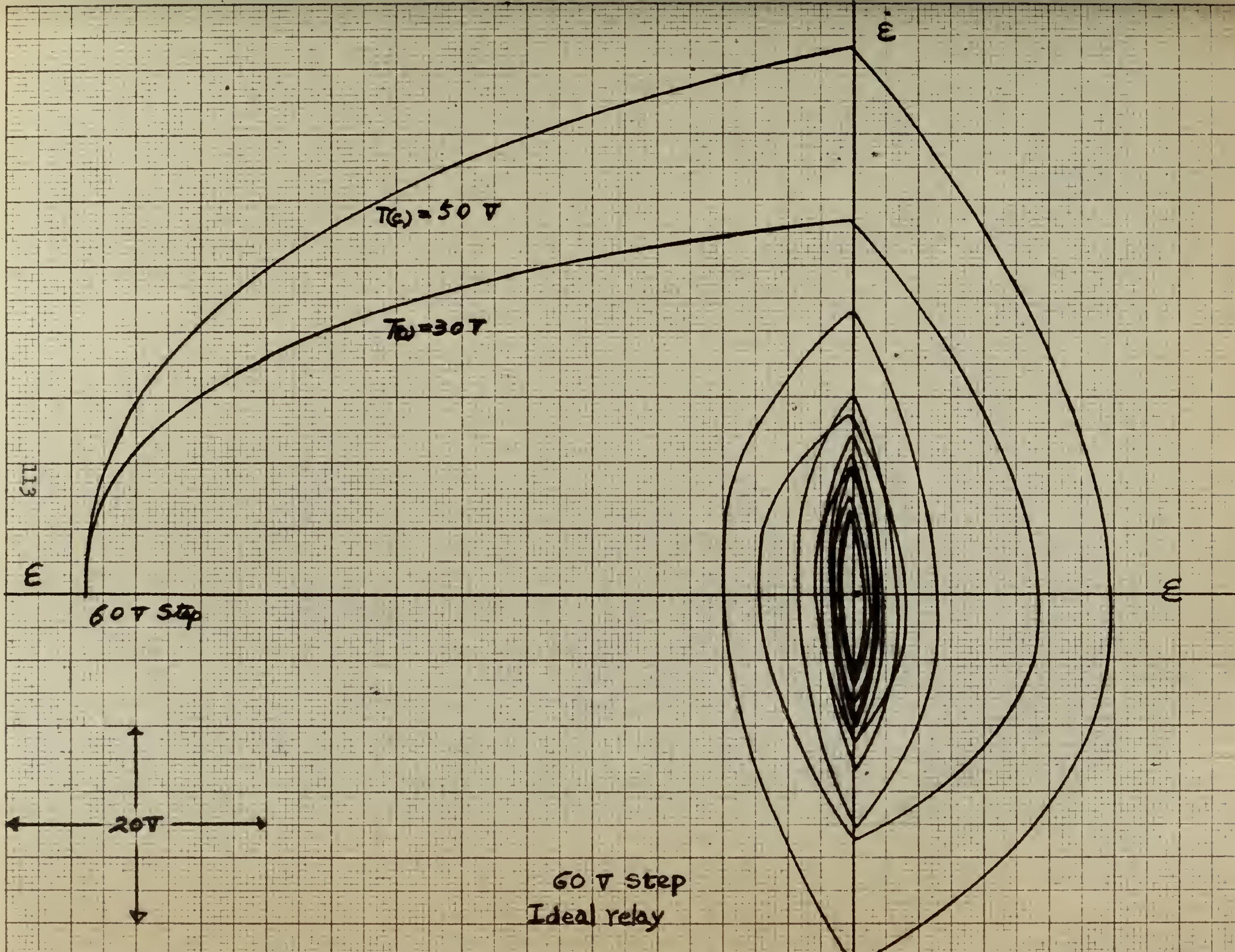


Fig. 6-5

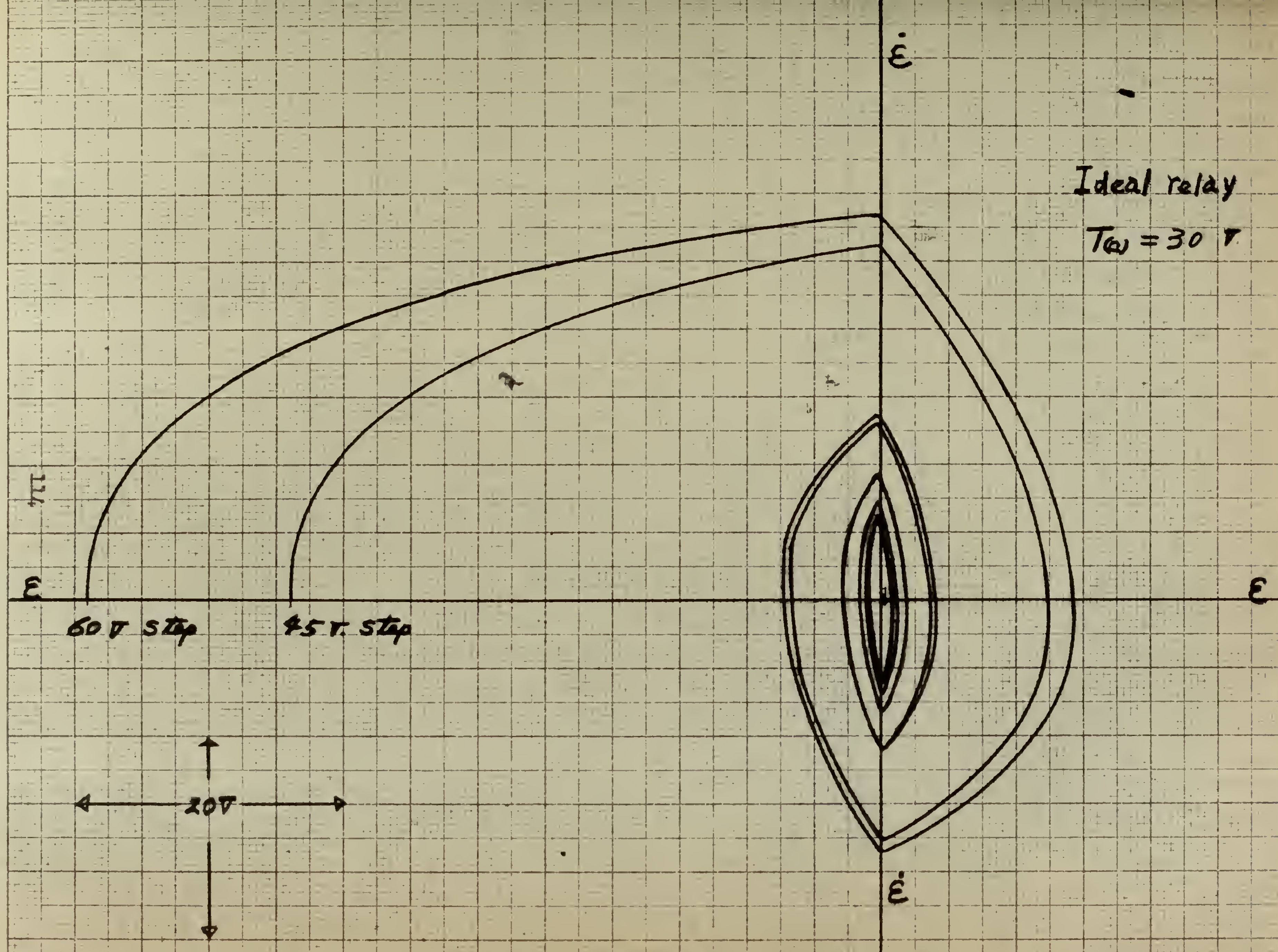
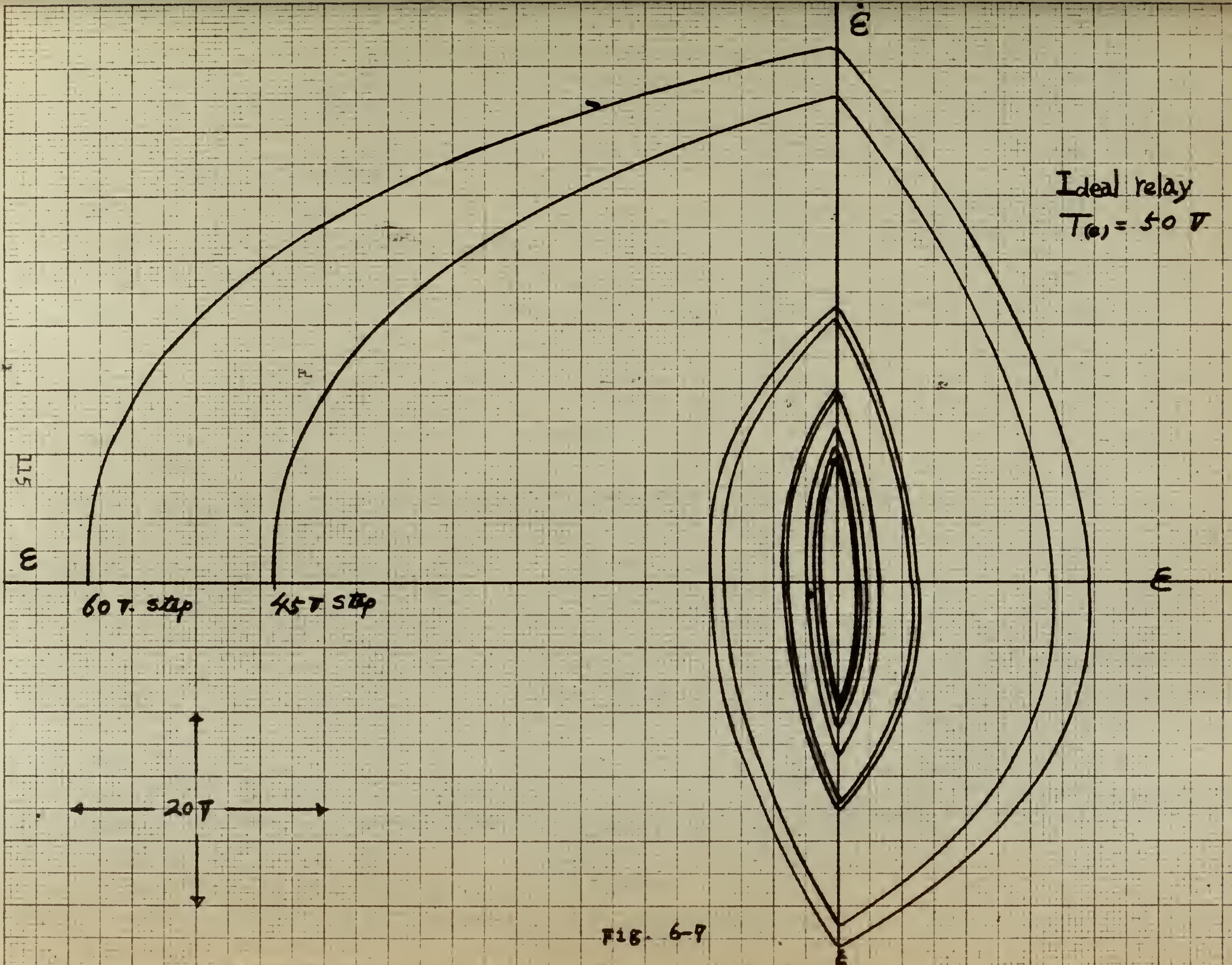
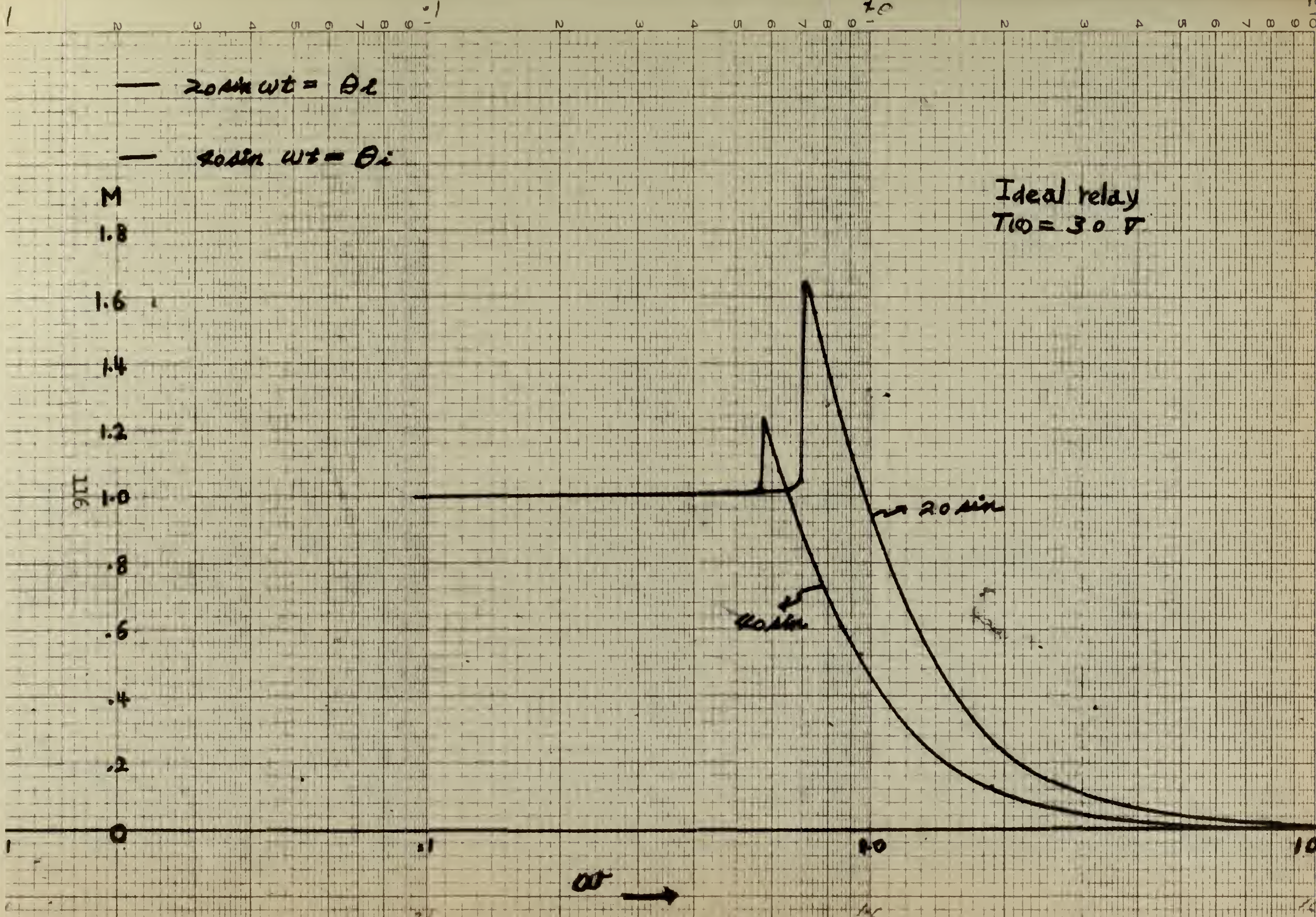

Ideal relay
$T_{(a)} = 30$ v
$\dot{\varepsilon}$
$\dot{\varepsilon}$
ε
ε
60 v step
45 v. step
20v

Fig. 6-6



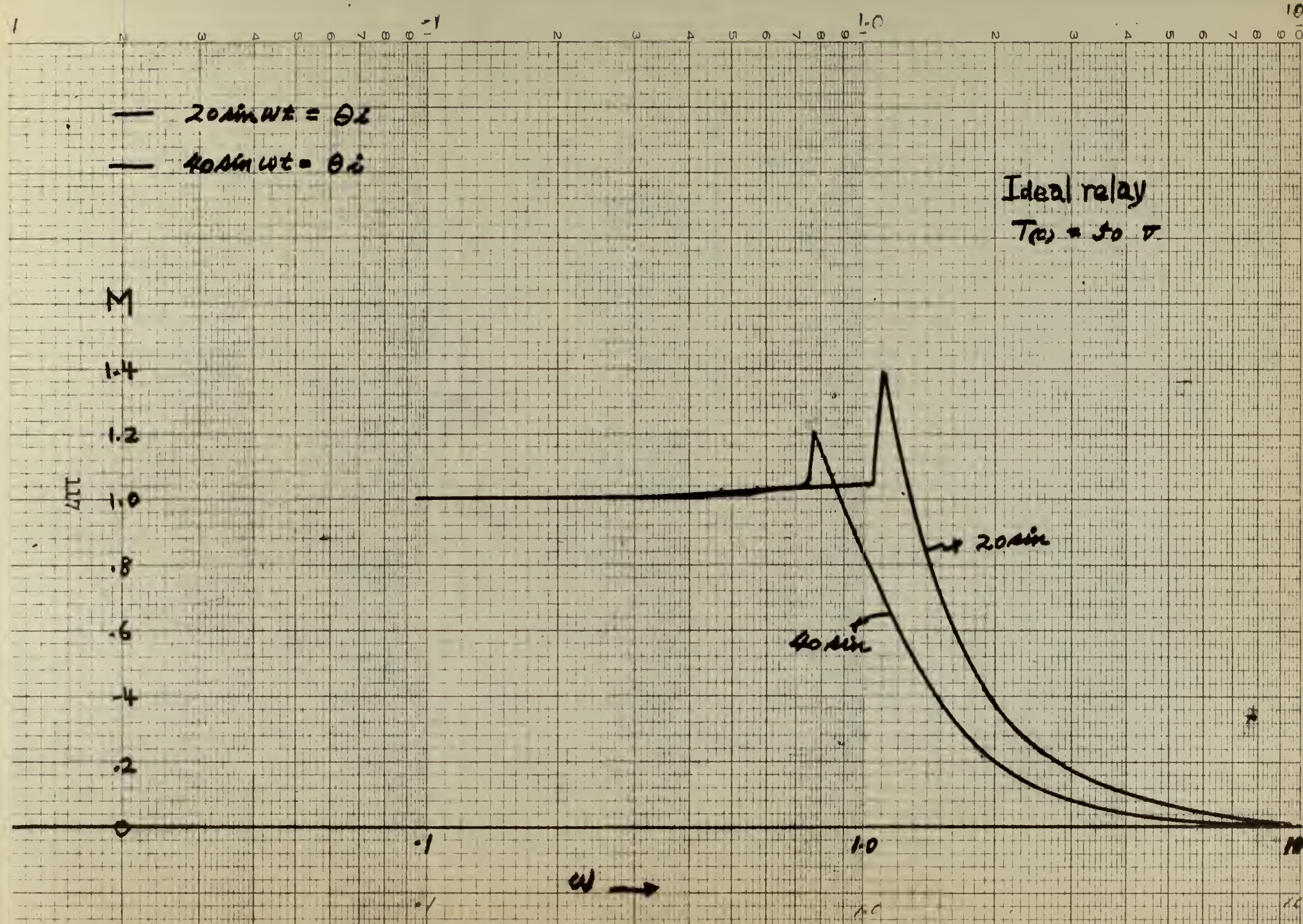
$\dot{\varepsilon}$
Ideal relay
$T_{(e)} = 50$ V
ε
60 v. step
45 v. step
20 V

Fig. 6-9



20 sin ωt = θl
40 sin ωt = θi
M
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
Ideal relay
T(ω) = 30 V
20 sin
40 sin
.1
1.0
10
ω →

Fig. 6-8



20 sin ωt = θi
40 sin ωt = θi
Ideal relay
T(e) = 50 v
M
1.4
1.2
1.0
.8
.6
.4
.2
0
.1
1.0
10
20 sin
40 sin
ω

FIG. 6-9

G. Relay with dead zone.

(a) Block diagram (Fig. 7-1)

The gain of amplifier Ka has only one effect on the system. It determines the sensitivity of relay dead space in terms of actual system position error. Thus, if the relay dead space is referred to a dead space in terms of error, Ka can be omitted. This implies that if the dead space simulation cirucit is placed before the amplifier with a proper scaling, the amplifier to simulate Ka can be omitted.

Let $D' \triangleq$ width of dead space of relay in relay voltage.

$D \triangleq$ width of dead space of relay in terms of position.

$T = .45$

$K_a = 20$

The system equations become;

$$J\ddot{\theta} + f\dot{\theta} = T \qquad \varepsilon > \frac{D}{2}$$
$$J\ddot{\theta} + f\dot{\theta} = -T \qquad \varepsilon < -\frac{D}{2}$$
$$J\ddot{\theta} + f\dot{\theta} = 0 \qquad -\frac{D}{2} < \varepsilon < \frac{D}{2}$$

(b) Scaling.

Let

$$D = \frac{D'}{K_a}$$

$$\alpha T = \alpha \theta = .015$$

$$\bar{T} = 30\,v.$$

Since D is in terms of error

i $D' = .9 \qquad D = .045 \qquad \bar{D} = 3\,v$

ii $D' = 1.8 \qquad D = .09 \qquad \bar{D} = 6\,v$

(c) Simulation circuit (Fig. 7-2).

Where $R_1 = 1.00\ M$

$r = 100\,K$

$E_1 = 40\,v.$

$$E_2 = 200V$$

$$R_0 \approx 10M$$

$$a = .15$$

gain of limiter amplifier $= \dfrac{R_0}{R_1 a} \approx 65$

$$\frac{b_1 E_2}{1-b_1} = \mp \qquad b_1 = .1304$$

$$\frac{b_1 r}{R_1} \approx .01$$

$$\text{i} \quad \frac{\bar{D}}{2} = \frac{3}{2} = \frac{a_1}{1-a_1} E_1 \qquad a_1 = .03613 \quad \text{for } \bar{D} = 3$$

$$\text{ii} \quad \frac{\bar{D}}{2} = \frac{6}{2} = \frac{a_1}{1-a_1} E_1 \qquad a_1 = .0698 \quad \text{for } \bar{D} = 6$$

$$\frac{\dfrac{b_1 r}{1 + \dfrac{b_1 r}{R_0}}}{R_1} \approx \frac{b_1 r}{R_1} \cong .01$$

$$X_0 \approx \pm \frac{b_1 E_2}{1-b_1} = \mp \qquad 3 < \bar{\varepsilon} < -3$$

(d) Resultant curves.

With dead space in the relay, system becomes stable. Wider the dead zone, the more stable system is obtained. These effects are shown in Fig. 7-3 and 7-4 of transientresponse curves. Because of noise amplified in amplifier, the limiter loop gain could not be gotten higher than 65 which is a little too small. Even with this gain, noises showed during the transition of dead space. Also in transient curves, drift of output position due to the noise effect is observed.

In the phase plane plot of Fig. 7-5, two plots for different size of nonlinearity seems to fall on top of each other. This is due to

similar isoclines of phase plane before and after the dead zone. As can be seen, two plots depart perceptively after a few cycles.

In frequency response curves, the first part of the curve up to the resonance peak is not a true one because the output did not reach the steady state condition.

(e) Discussion.

Due to not high enough loop gain of 65, the simulation circuit imposes additional error in the simulation. For error voltages less than $\bar{\varepsilon} < \frac{30}{65} \pm \frac{\bar{D}}{2}$ region, the torque applied is not $\bar{T} = 30$ but $(\bar{\varepsilon} - \frac{\bar{D}}{2})\,65$. This error can be reduced if the gain can be increased. If batteries or low noise voltage sources were used, gain may be gotten higher. The term $\frac{b_1 r}{R_1} \approx .01$ imposes another error in the circuit. If $\bar{\varepsilon}_{max} = 60$, the simulation will have 1.8% error in the torque. But this error factor can be reduced with the use of higher voltage sources or lower resistance potentiometer.

Frequency response data
(relay with dead zone)

$\bar{D} = 3$

		$\bar{\theta}_i = 20 \sin \omega t$		$\bar{\theta}_i = 40 \sin \omega t$	
$\bar{f}$	ω	$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	20.5	1.025	40	1
.03	.283	20.7	1.035	41	1.025
.04	.377	21	1.05	41.7	1.043
.05	.472	21.8	1.09	41.7	1.043
.055	.518			53.3	1.333
.057	.537			51	1.275
.06	.566	22	1.1	47	1.175
.07	.66	22	1.1	37	.925
.08	.754	22.8	1.14	29.5	.738
.081	.763	28.5	1.425		
.083	.782	27.5	1.375		
.09	.848	23.8	1.19		
.1	.943	19.3	.965	19.8	.495
.2	1.885	5	.25	4.8	.12
.4	3.77	1.65	.0875	1.15	.0288
.6	5.66	.5	.025	.5	.0125
1.0	9.43	.13	.0065	.11	.00275

Frequency response data
(relay with dead zone)

$\bar{D} = 6$

$\bar{f}$	ω	$\bar{\theta}_i = 20 \sin \omega t$		$\bar{\theta}_i = 40 \sin \omega t$	
		$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	20	1.0	40	1.0
.03	.283	20	1	41	1.025
.04	.377	21	1.05	41.5	1.038
.05	.472	21.4	1.07	43.5	1.088
.06	.566	22.2	1.11	48	1.2
.065	.613	23.2	1.16	54	1.35
.07	.66	23.5	1.175	37.7	.943
.075	.707	32.5	1.625		
.08	.754	29	1.45	30	.75
.1	.943	19	.95	20	.5
.2	1.885	5	.25	5	.125
.4	3.77	1.2	.06	1.1	.0275
.6	5.66	.5	.025	.5	.0125
.8	7.54	.24	.012	.23	.00575
1.0	9.43	.1	.005	.12	.003

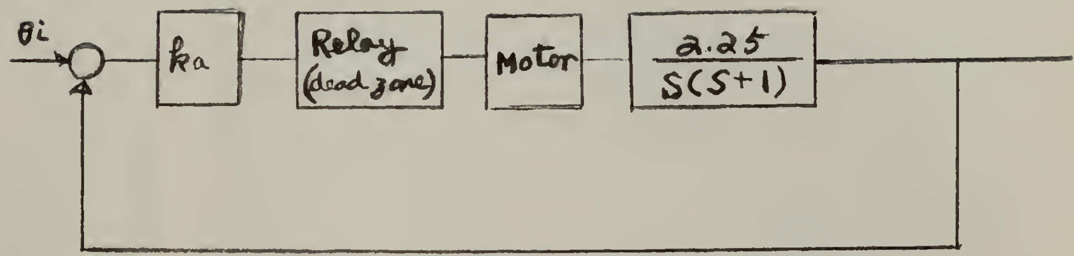


Fig. 7-1 Block diagram of a relay servo system with dead zone.

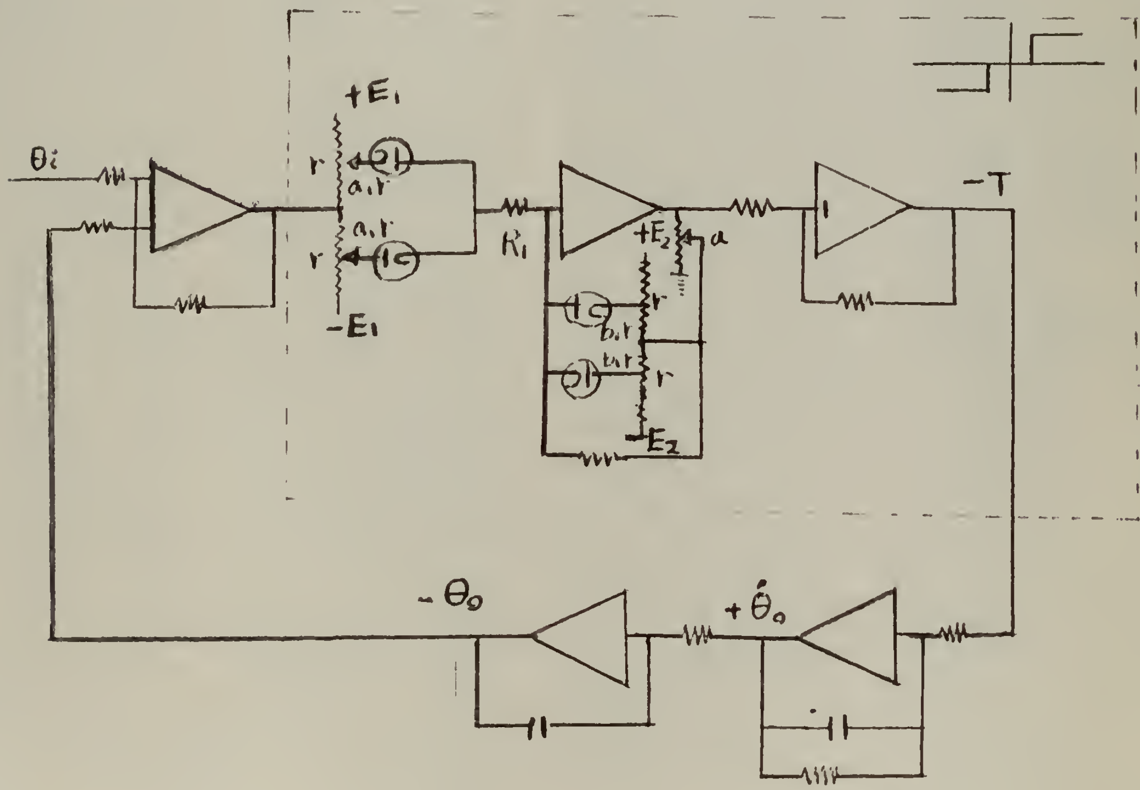
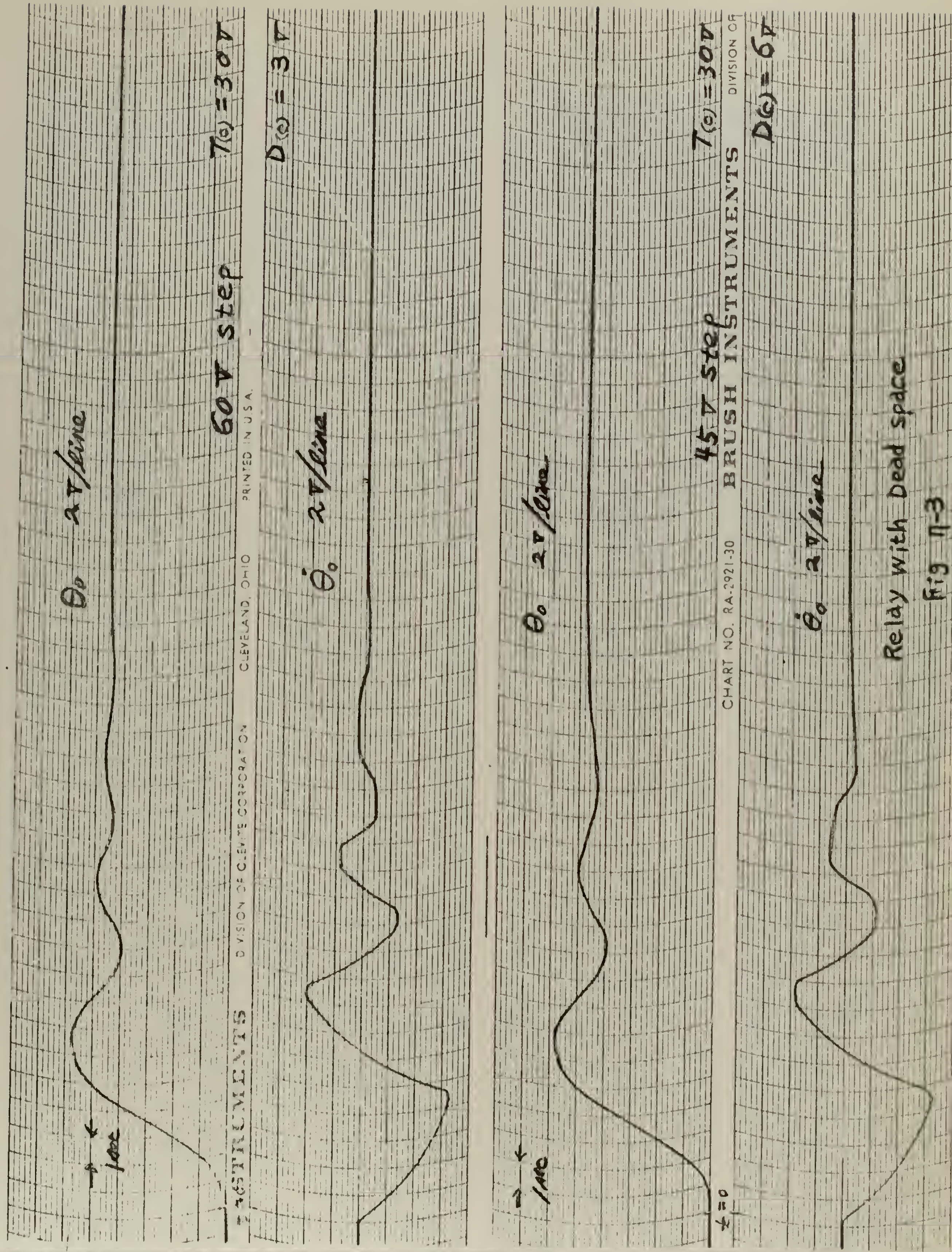
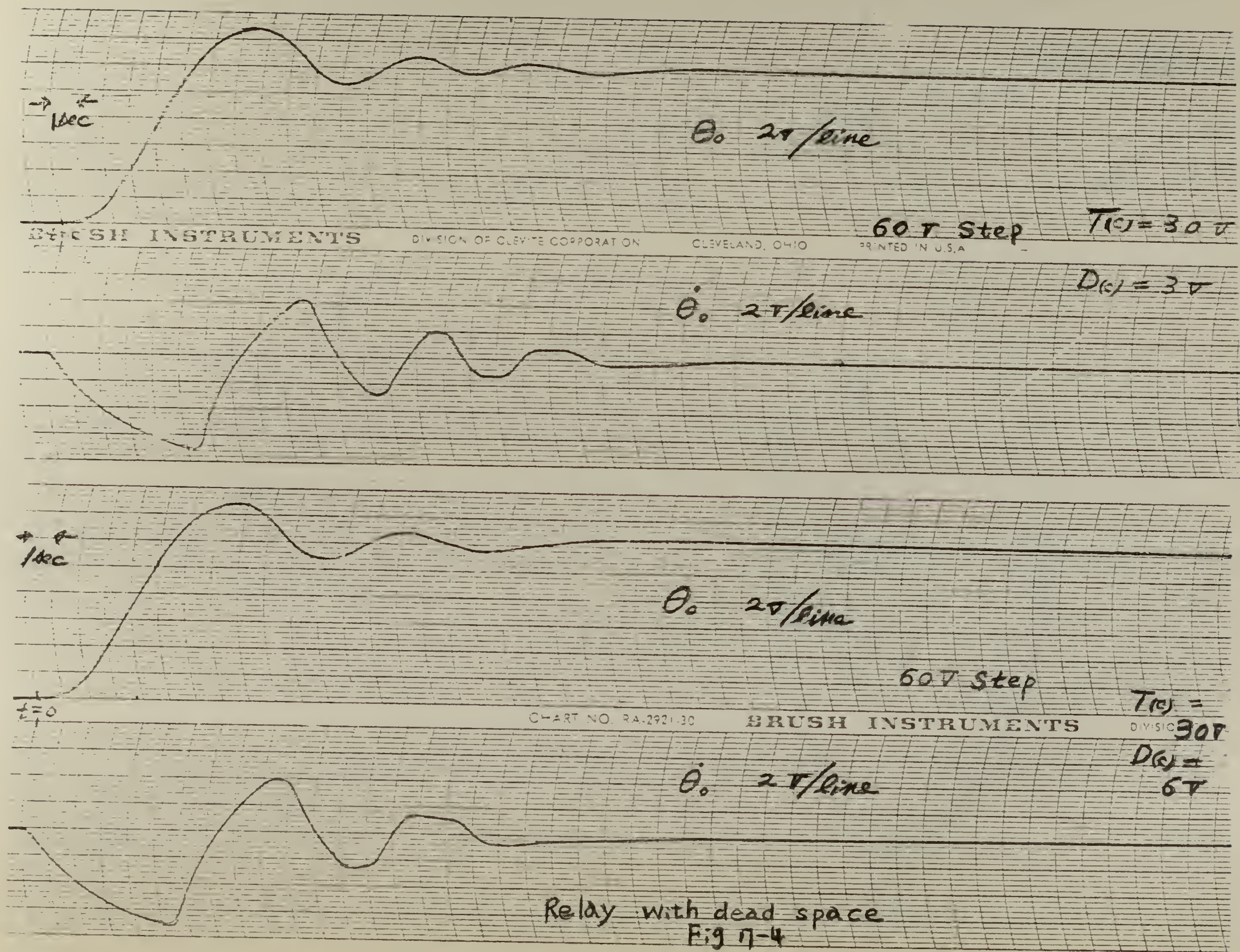


Fig. 7-2 Simulation circuit for the system shown in Fig. 7-1.



Relay with Dead Space
Fig 11-3



Relay with dead space
Fig 7-4

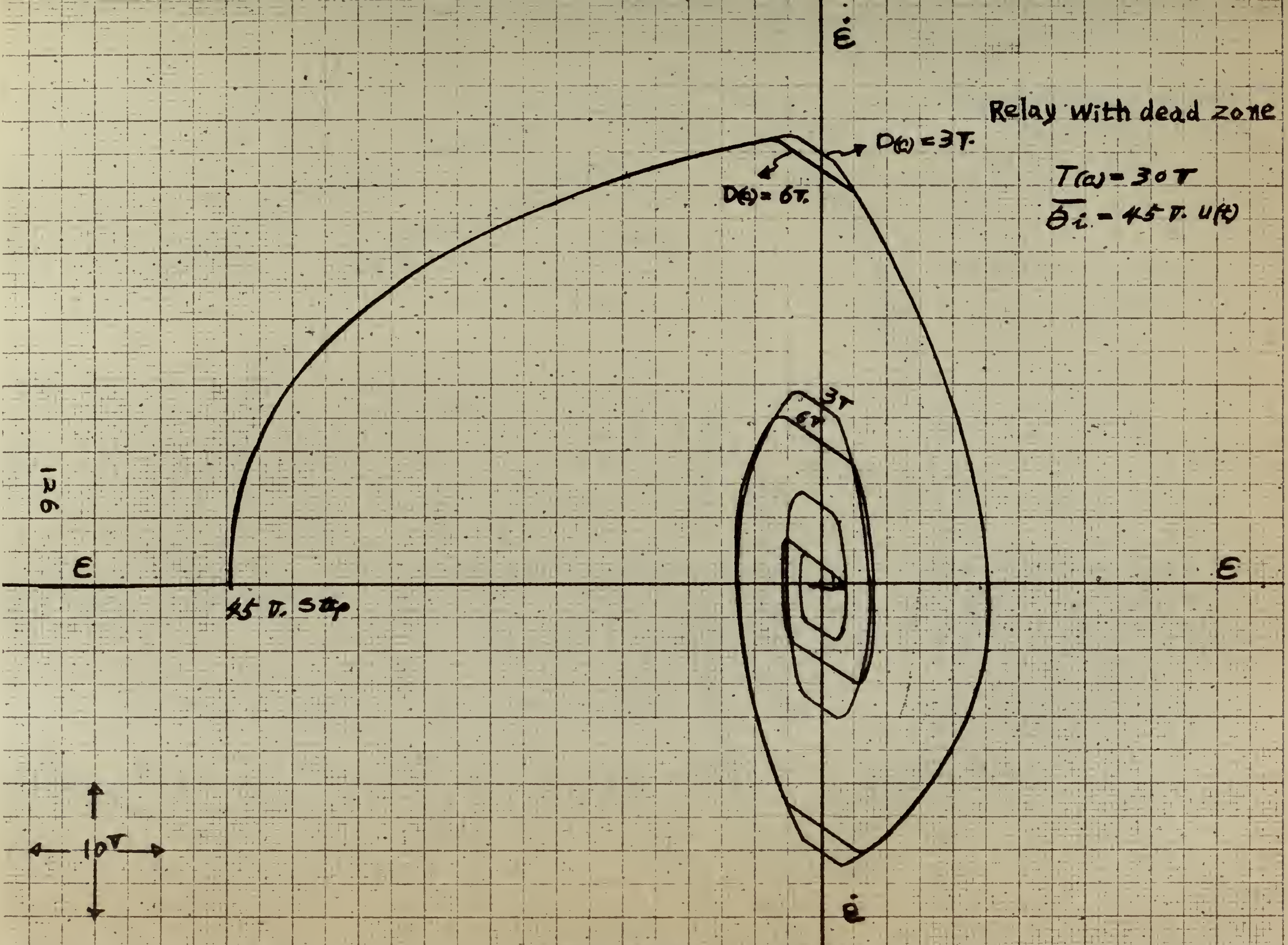

Relay with dead zone
$D(t) = 3T.$
$D(t) = 6T.$
$T(a) = 30T$
$\overline{\theta_i} = 45T.\ u(t)$
$\dot{\varepsilon}$
ε
ε
45 T. Step
3T
6T
10^T

Fig. 7-5

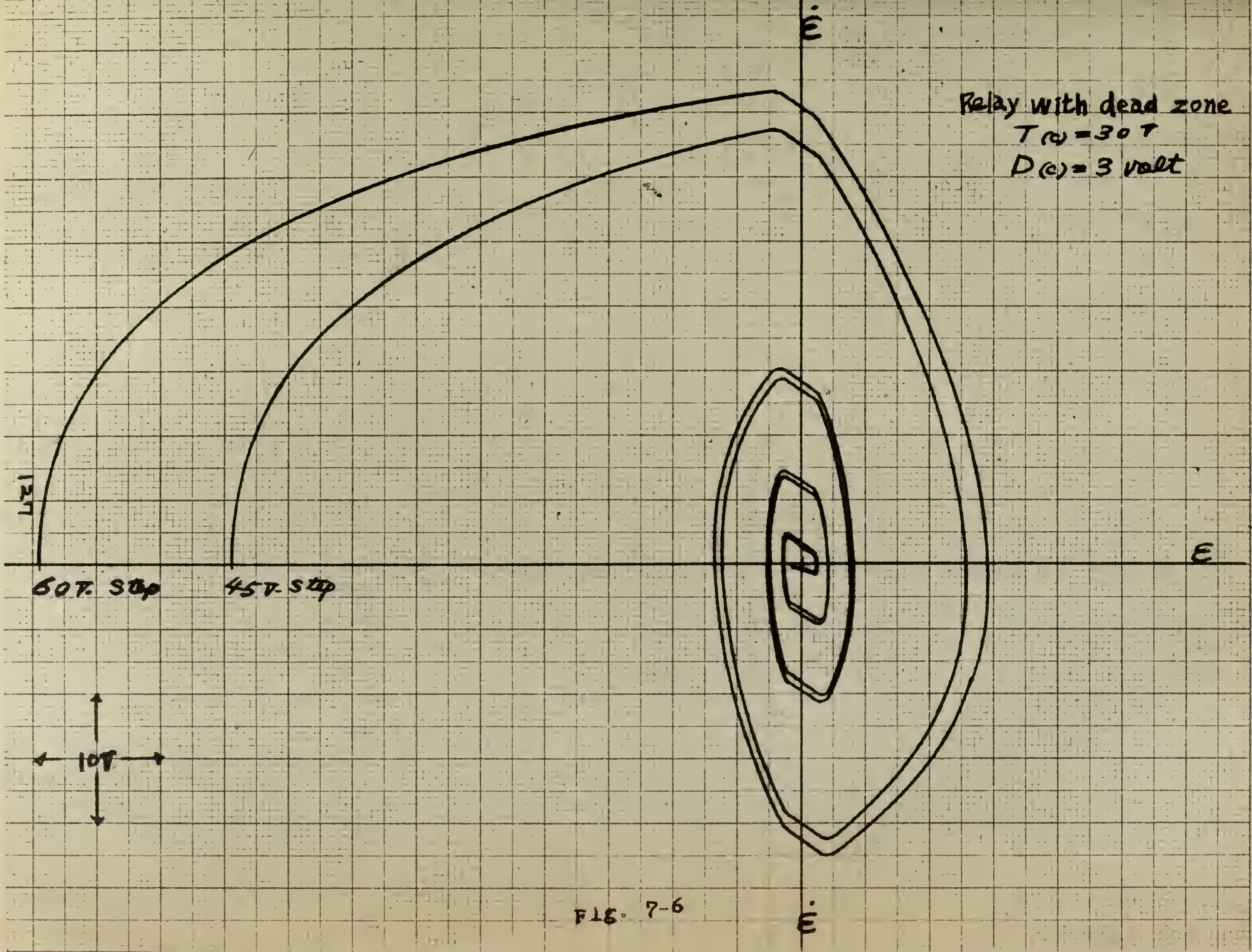


Fig. 7-6

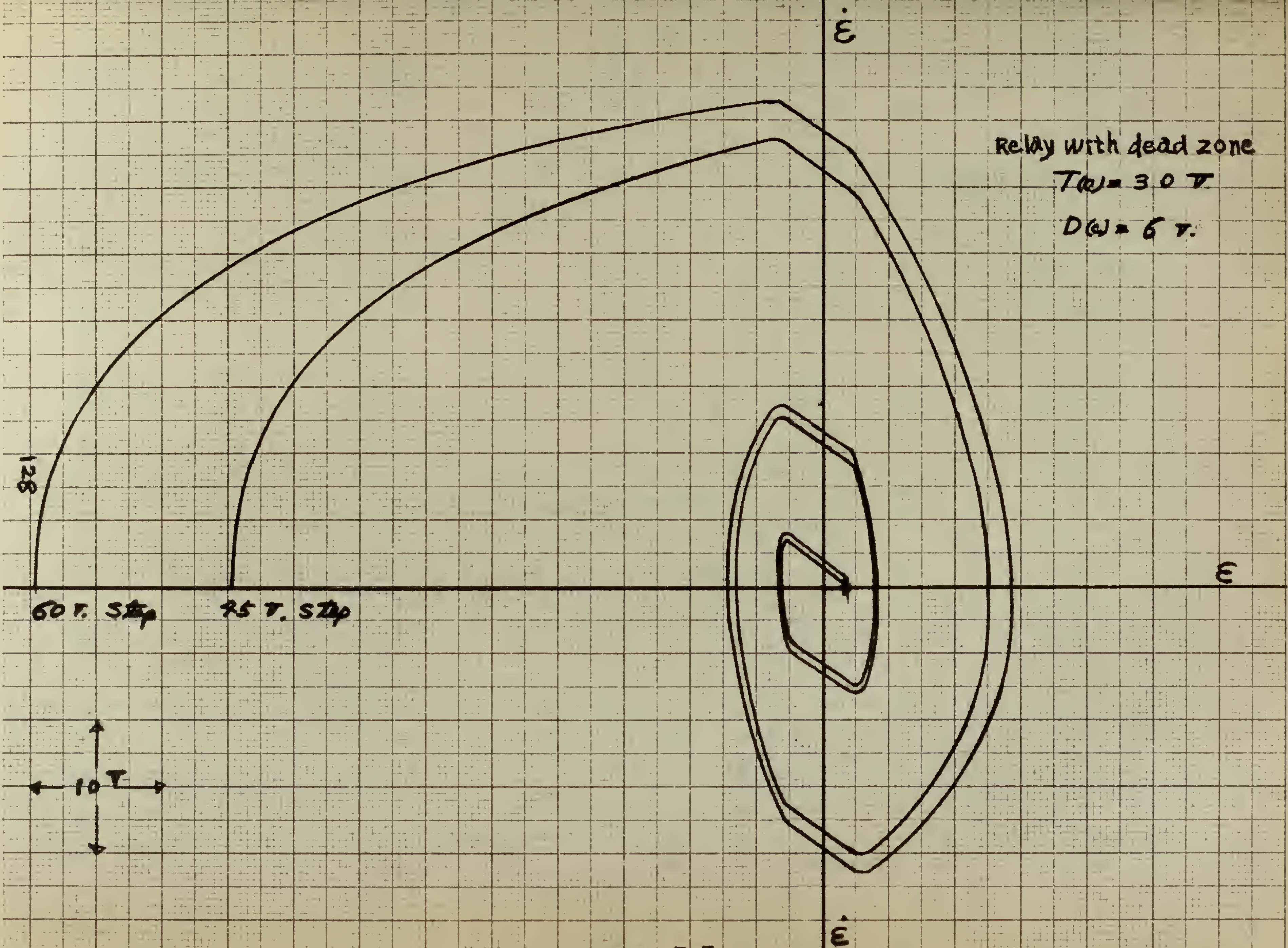
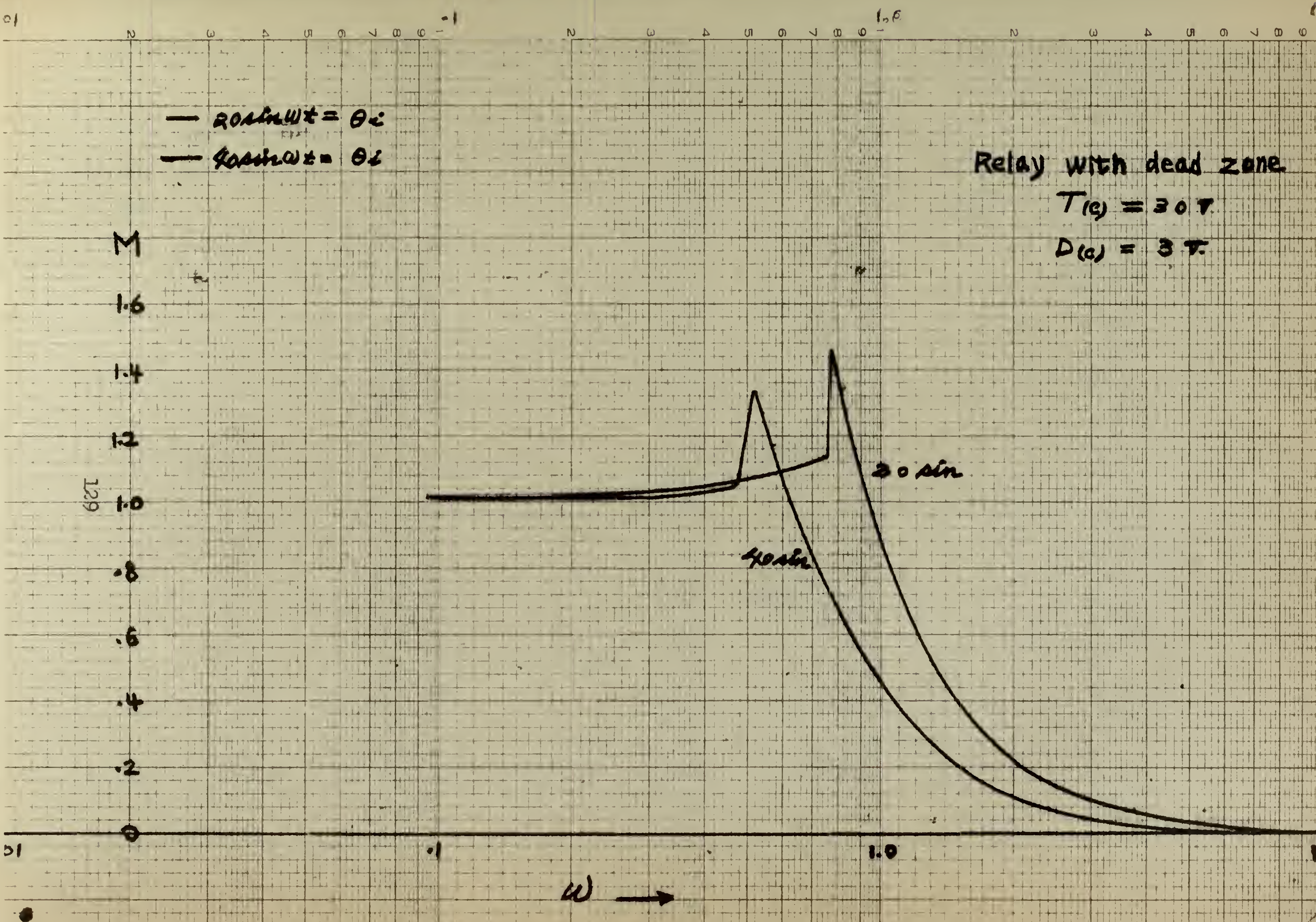
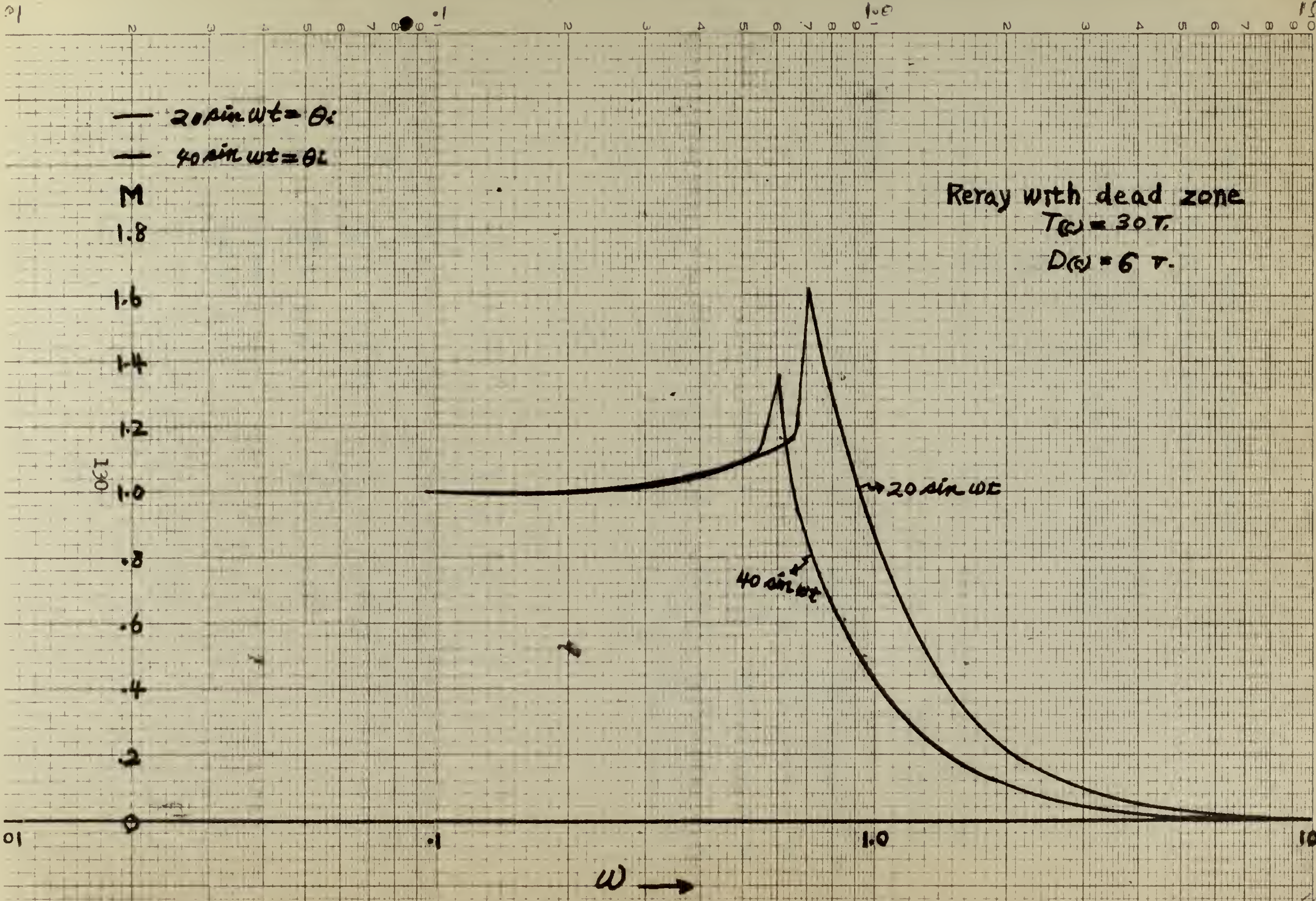

Relay with dead zone
T(a) = 3.0 v.
D(a) = 6 v.
$\dot{\varepsilon}$
ε
60 v. Step
45 v. Step
10 v.

Fig. 7-7



Relay with dead zone
$T_{(c)} = 30\,V.$
$D_{(c)} = 3\,V.$
$20 \sin \omega t = \theta_i$
$40 \sin \omega t = \theta_i$
M
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
30 sin
40 sin
.1
1.0
10
$\omega \longrightarrow$

Fig. P-8



— 20 sin ωt = θi
— 40 sin ωt = θi
M
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
Reray with dead zone
T(c) = 30 T.
D(c) = 6 T.
20 sin ωt
40 sin ωt
.01
.1
1.0
10
ω →

Fig. 7-9

H. Relay with dead space and hysteresis.

(a) Block diagram (Fig. 8-1).

Let; $\frac{p'}{2} \triangleq$ Relay pull in voltage.

$\frac{d'}{2} \triangleq$ relay drop out voltage.

$\frac{p}{2} \triangleq \frac{p'}{2}$ referred to error.

$\frac{d}{2} \triangleq \frac{d'}{2}$ referred to error.

The amplifier gain Ka has same effect as was discussed in the relay with dead space, and is treated in same manner. Then system equations become:

$$J\ddot{\theta} + f\dot{\theta} = 0 \qquad \varepsilon < \frac{p}{2} \qquad |\varepsilon| \text{ increase in } +$$

$$J\ddot{\theta} + f\dot{\theta} = T \qquad \varepsilon > \frac{p}{2} \qquad |\varepsilon| \text{ increase in } +$$

$$J\ddot{\theta} + f\dot{\theta} = 0 \qquad \varepsilon < \frac{d}{2} \qquad |\varepsilon| \text{ decrease in } +$$

$$J\ddot{\theta} + f\dot{\theta} = -T \qquad \varepsilon < \frac{p}{2} \qquad |\varepsilon| \text{ increase in } -$$

$$J\ddot{\theta} + f\dot{\theta} = 0 \qquad \varepsilon > \frac{d}{2} \qquad |\varepsilon| \text{ decrease in } -$$

Let;

$T = .67$

$k_a = 20$

i.

$\frac{p'}{2} = 1.25 \text{ v.} \qquad \frac{p}{2} = .075 \text{ v.}$

$\frac{d'}{2} = .9 \text{ v.} \qquad \frac{d}{2} = .045 \text{ v.}$

ii.

$\frac{p'}{2} = 1.25 \text{ v.} \qquad \frac{p}{2} = .075 \text{ v.}$

$\frac{d'}{2} = .3 \text{ v.} \qquad \frac{d}{2} = .015$

(b) Scaling

$\bar{T} = 44T.$

$\because$ 44 volt dry battery was available.

i.

$$\frac{\bar{p}}{2} = \frac{p}{2}\frac{1}{d\theta} = 5 \text{ v.}$$

$$\bar{d} = \frac{d}{2}\frac{1}{d\theta} = 3 \text{ v.}$$

$$\frac{\bar{p}}{2} - \frac{d}{2} = 3$$

ii.

$$\frac{\bar{p}}{2} = \frac{p}{2}\frac{1}{d\theta} = 5 \text{ v.}$$

$$\frac{\bar{d}}{2} = \frac{d}{2}\frac{1}{d\theta} = 1 \text{ v.}$$

$$\frac{\bar{p}}{2} - \frac{\bar{d}}{2} = 4 \text{ v.}$$

(c) Simulation circuit (Fig. 8-2).

where;

$$R_1 \approx .1 \text{ M}$$

$$R_0 \approx 20 \text{ M}$$

$$E_1 = 100 \text{ v.}$$

$$a_1 = .0476 \qquad \therefore \quad \frac{a_1}{1-a_1} E = \frac{\bar{p}}{2} = 5 \text{ v.}$$

i. $f = .0454$ for $\frac{\bar{p}}{2} - \frac{\bar{d}}{2} = 2 \text{ v.}$

$$\frac{\bar{p}}{2} - \frac{\bar{d}}{2} = f \, E_2$$

$$f = \frac{2}{44} = .0454$$

ii. $f = .091$ for $\frac{\bar{p}}{2} - \frac{\bar{d}}{2} = 4 \text{ v.}$

$$f = \frac{4}{44} = .091$$

limiter loop gain $= \frac{R_0}{R_1} \approx 200$

(d) Resultant curves.

The case of $\frac{p}{2} - \frac{d}{2} = 0$; ie. a relay with dead space only were included in this paragraph for comparison purposes, because $\bar{T}$ is different from the one used in paragraph F of this section.

In the transient response curves, comparing Fig. 8-3 and 8-4 to

Fig. 8-5 (relay with dead zone only), it is evident, the relay with hysteresis has more oscillation with higher overshoot. This implies that the effect of hysteresis in relay servo is a destabilization. For cases of hysteresis chosen, it exhibits a definite limit cycle, while if $\frac{p}{2} - \frac{d}{2} = 0$, system is stable.

Fig. 8-6 shows three cases of nonlinearities. If there is no hysteresis effect in the relay, it zeros in to the origin as shown by curve "a", while others reach limit cycle.

The first part of frequency response curves up to the resonance peak is an approximation. The output did not reach the steady state in that part of the frequency. The purpose of these curves are to show the presence of the nonlinearity in the system.

(e) Discussion.

As was stated in Section 2, the limiter gain $\frac{R_o}{R_1}$ must be great in order for a satisfactory operation of this circuit. With rectified D-C power source in place of E2, gain could not be gotten higher than about 50. It was then necessary to replace E2 with dry batteries to rid it of noises. Dry battery available in the laboratory was 44 volt. For this reason, it was found necessary to change the torque to $\bar{T} = 44$ volt from previous case of relay servo simulation.

With the use of dry batteries, the gain could be increased to about 200 which gave satisfactory results.

In all phase plane plots, errors in "drop out" and "pull in" voltages is 12% in this simulation determined from the phase plane plots assuming that the plotter involves no error in itself. This error, is believed to be caused from inaccurate representation of $\frac{\bar{p}}{2}$ and the non-

ideal characteristics of diodes used.

Also, the loop gain of non-infinity introduces error. In general, if the power sources are noise free so that the loop gain can be increased to a higher value, and if accurate representation of "pull in" voltages can be accomplished, it could be reduced.

Or if the amount of error involved with the circuit is known, then the error can be corrected by adjusting the potentiometer setting "f".

Frequency response data
(relay with dead zone, hysteresis)

$\bar{T} = 44 \qquad \frac{\bar{p}}{2} = 5$

$\frac{\bar{p}}{2} - \frac{\bar{d}}{2} = 2\ V.$

$\bar{f}$	ω	$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	20.2	1.01	41	1.025
.03	.283	20.2	1.01	41	1.025
,04	.377	20.5	1.025		
.05	.472	21	1.05	44	1.1
.06	.566	21.5	1.075		
.07	.66	22.5	1.125	48	1.2
.073	.688			48.5	1.212
.075	.706			53	1.325
.076	.716			52	1.3
.08	.754	24	1.2	47.5	1.19
.09	.848	27	1.35	37.5	.938
.1	.943	32	1.6	31.5	.795
.11	1.035	27	1.35		
.2	1.885	8.8	.44	8.4	.21
.4	3.77	2.05	.1025	2	.05
.6	5.66	.9	.045	.9	.0225
.8	7.54	.5	.025	.5	.0125
1.0	9.43	.31	.0155	.3	.0075
		$\bar{\theta}_i = 20 \sin \omega t$		$\bar{\theta}_i = 40 \sin \omega t$	

Frequency response data
(relay with dead zone, hysteresis)

$\bar{T} = 44 \quad \frac{\bar{P}}{2} = 5$

$\frac{\bar{P}}{2} - \frac{d}{2} = 4$ V.

$\bar{f}$	ω	$\bar{\theta}_o$	M	$\bar{\theta}_o$	M
.01	.0943	20	1	40	1
.03	.283	21	1.05	41	1.025
.04	.377	22	1.1	43	1.075
.05	.472	23	1.15	44	1.1
.06	.566	24	1.2	45	1.125
.066	.622			64	1.6
.068	.642			61	1.525
.07	.66	26	1.3	59	1.475
.08	.754	28	1.4	46.5	1.162
.086	.81	40	2.0		
.088	.828	39	1.95		
.09	.848	37	1.85	38	.95
.1	.943	31	1.55	31	.775
.2	1.885	8.2	.41	8	.2
.4	3.77	2	.05	1.9	.0475
.6	5.66	.9	.045	.9	.0225
.8	7.54	.51	.0255	.5	.0125
1.0	9.43	.28	.014	.3	.0075
		$\bar{\theta}_i = 20 \sin \omega t$		$\bar{\theta}_i = 40 \sin \omega t$	

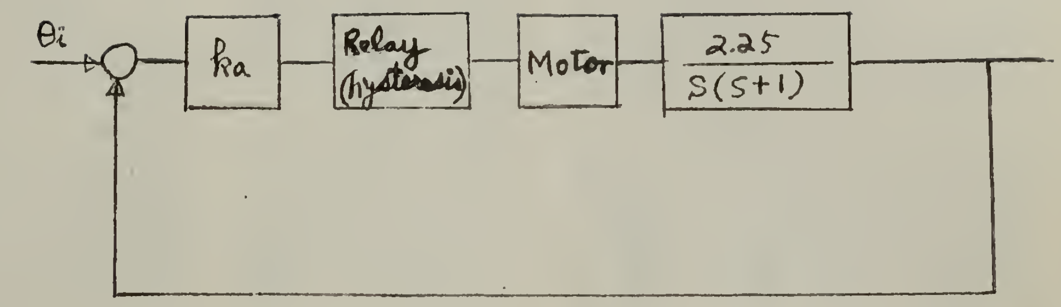


Fig. 8-1 Block diagram of a relay servo system with hysteresis effects in the relay.

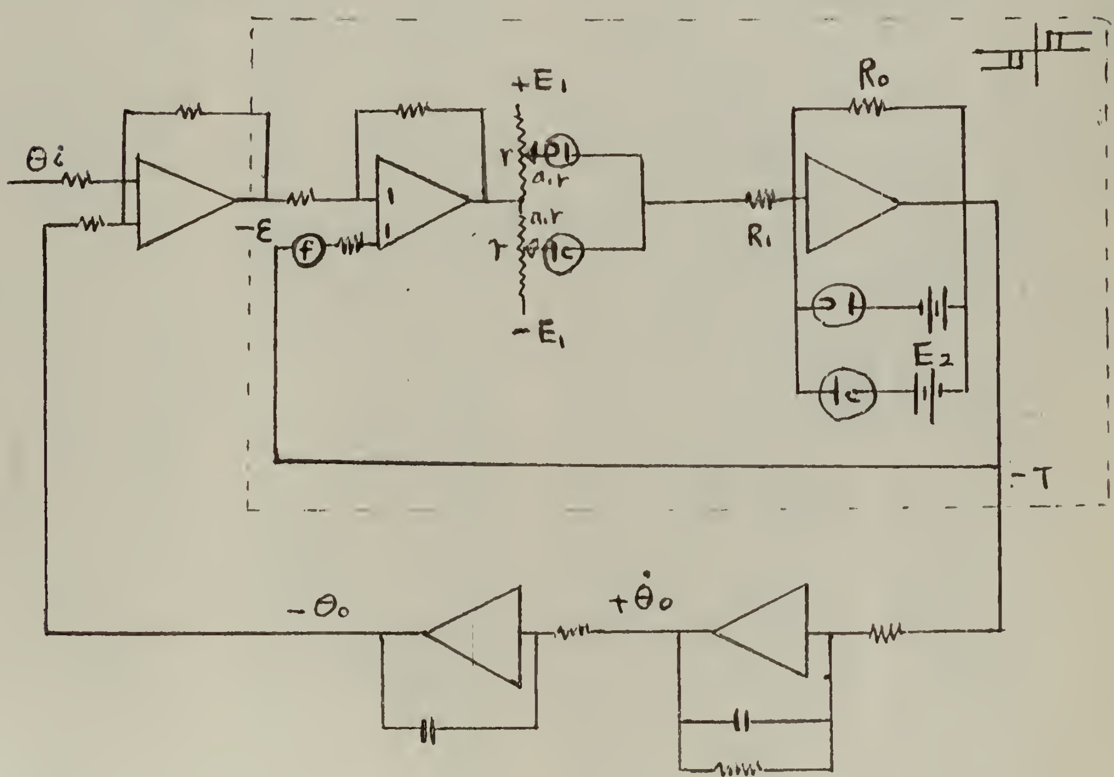
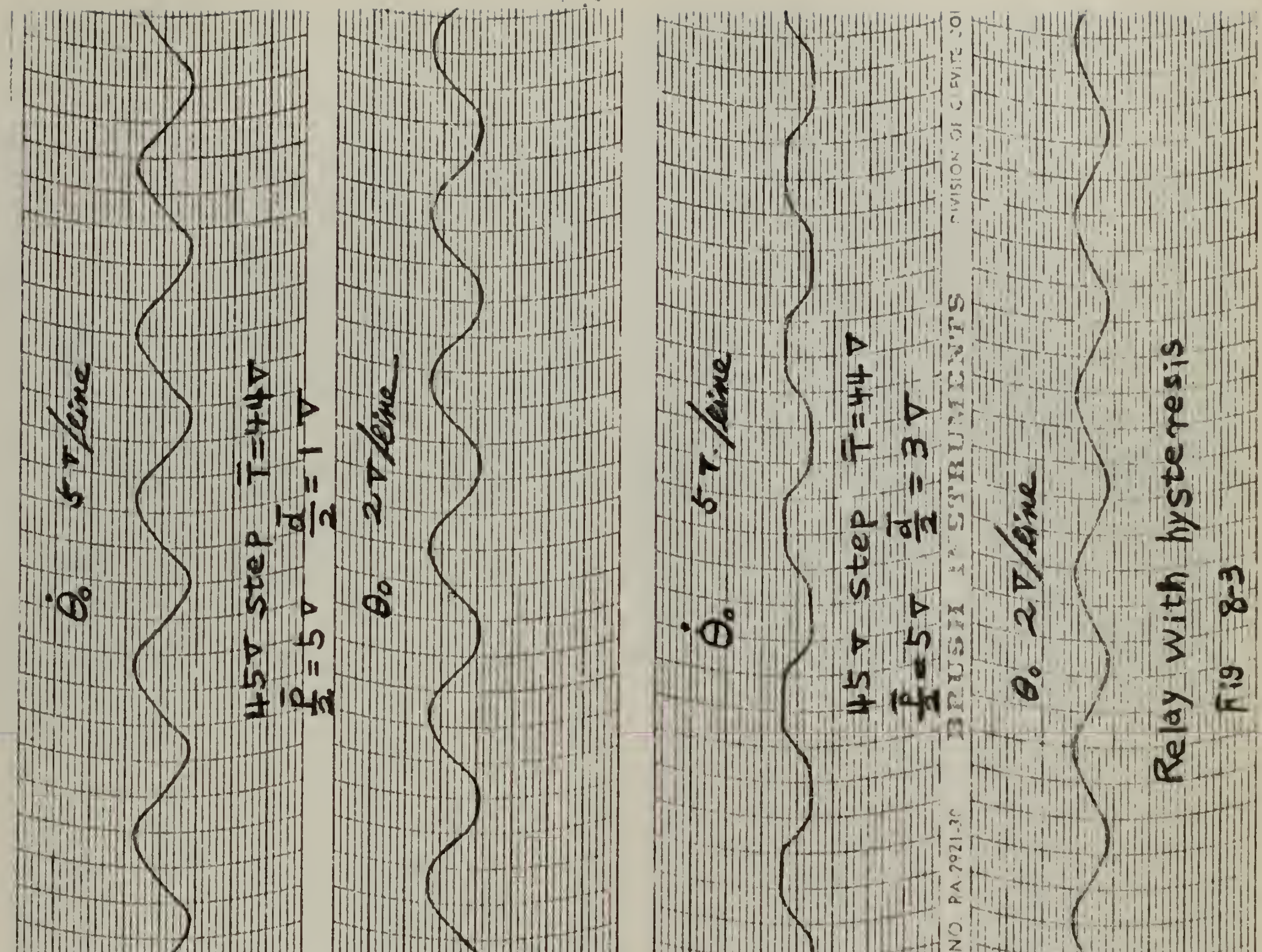
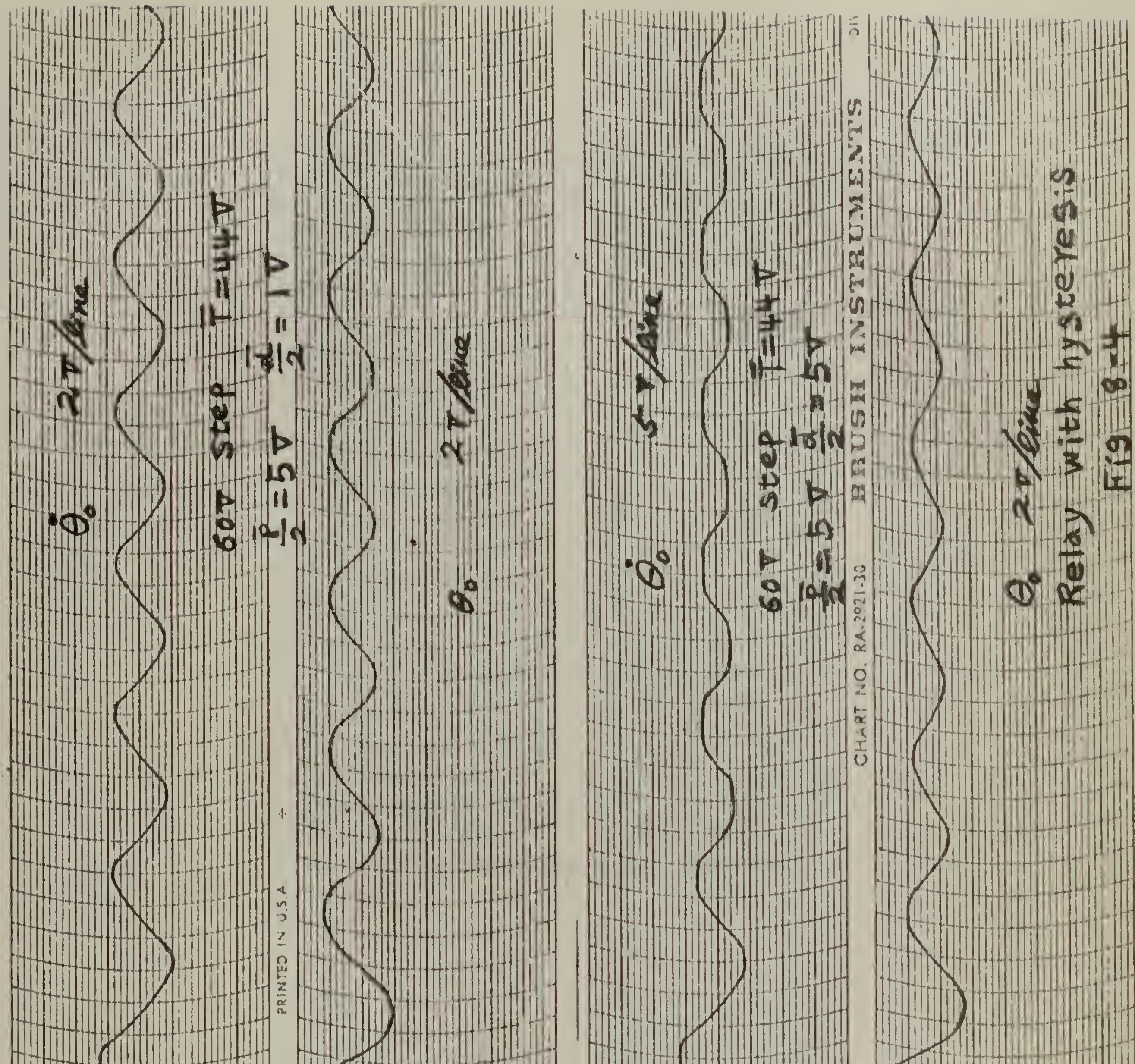


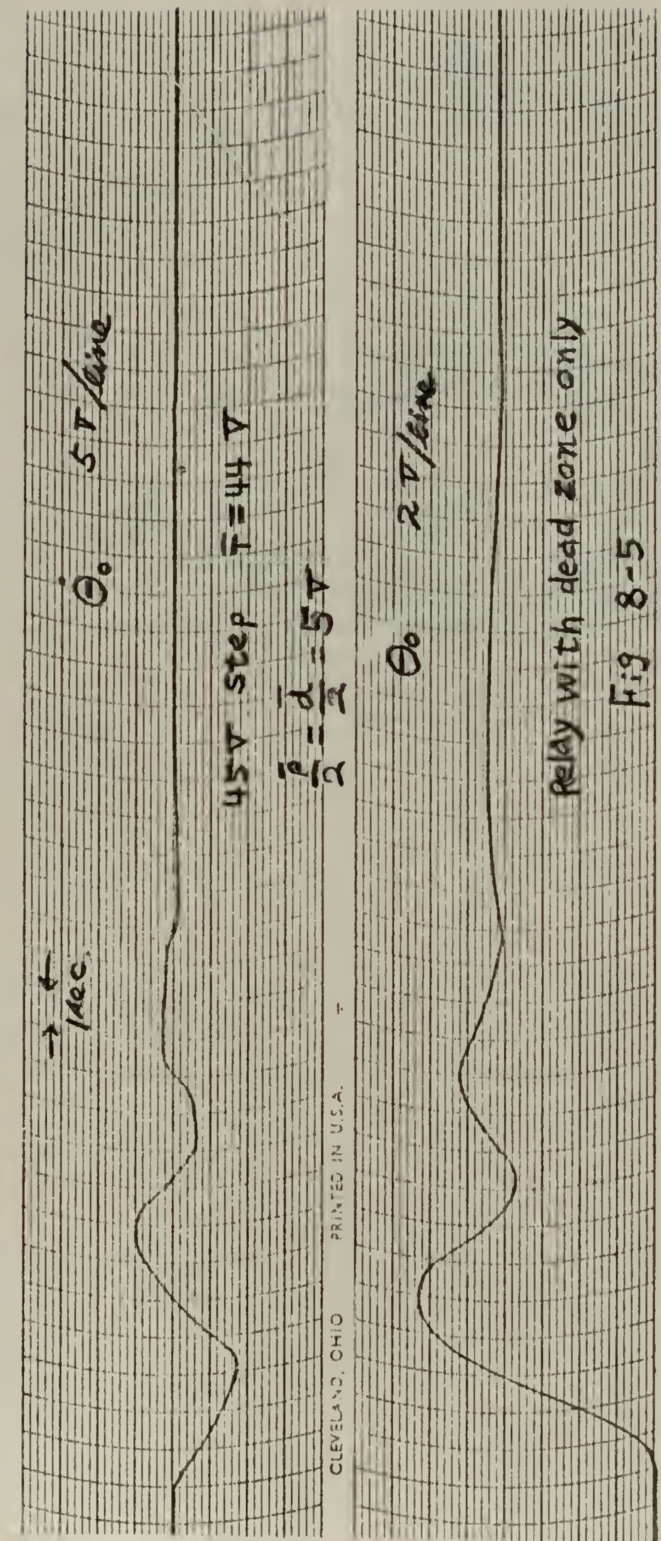
Fig. 8-2 Simulation circuit for the system shown in Fig. 8-1.



Relay with hysteresis
Fig 8-3



Relay with hysteresis
Fig 8-4



1 sec.
5 v/line
45 v step
2 v/line
Relay with dead zone only
CLEVELAND, OHIO
PRINTED IN U.S.A.

Fig 8-5

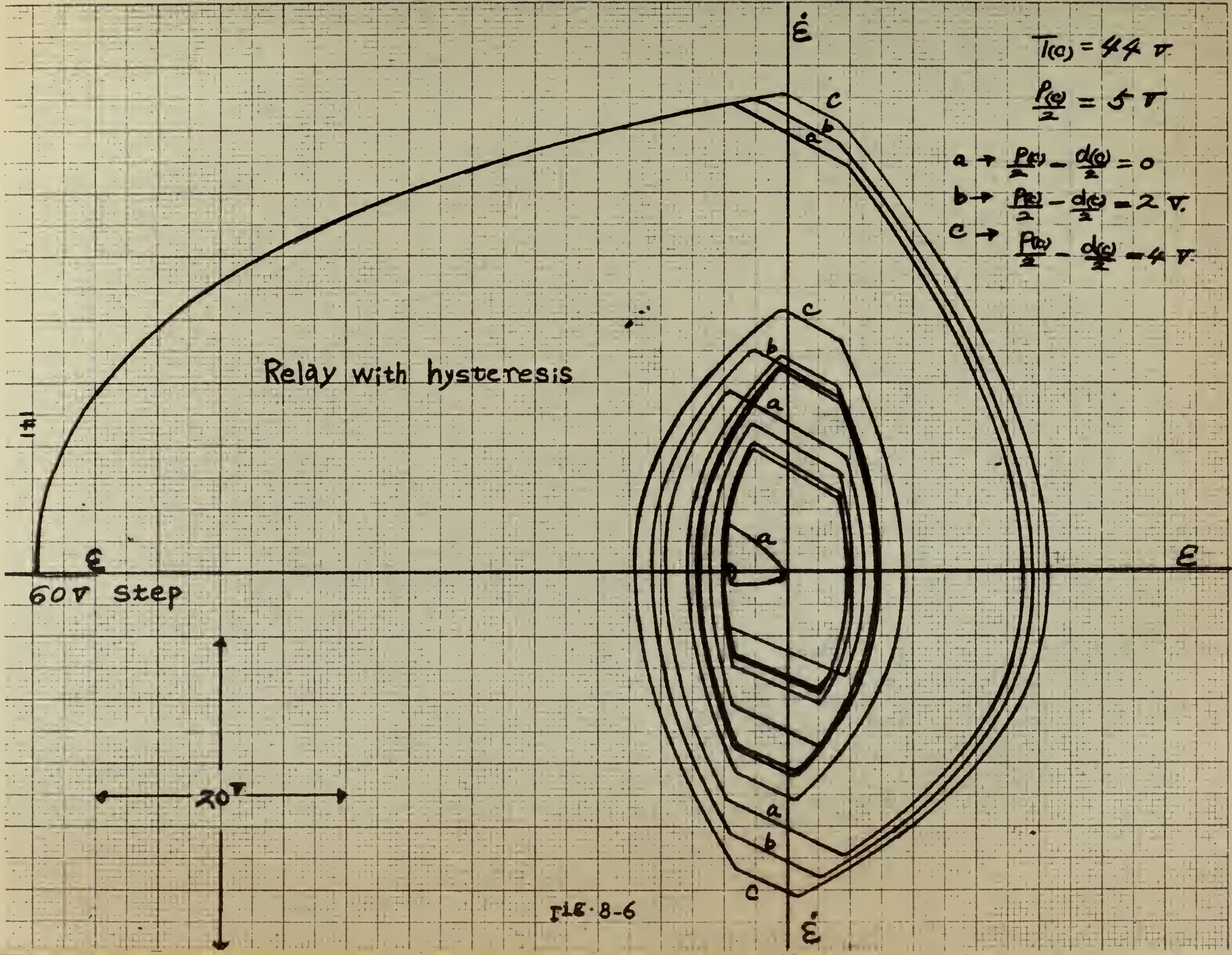
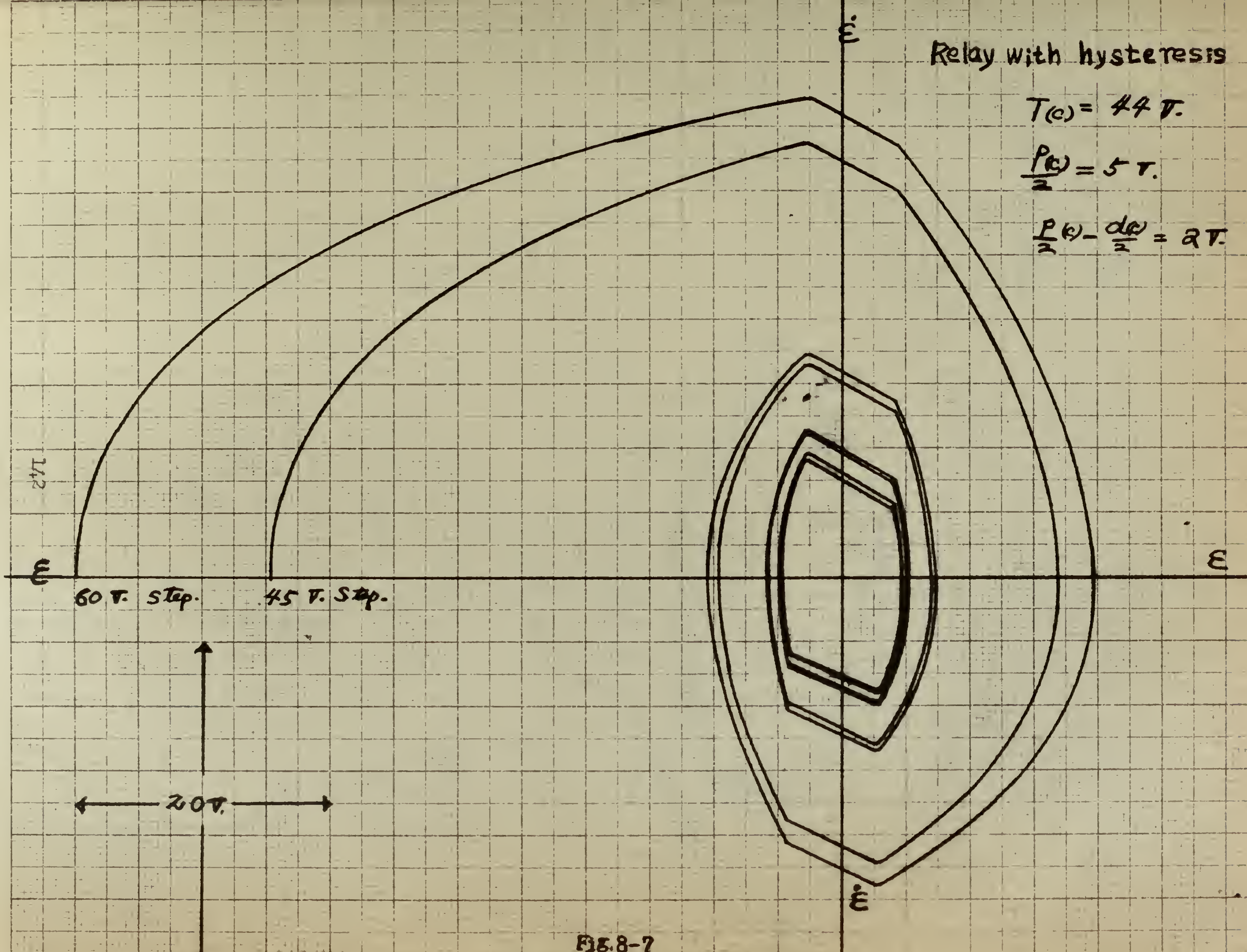
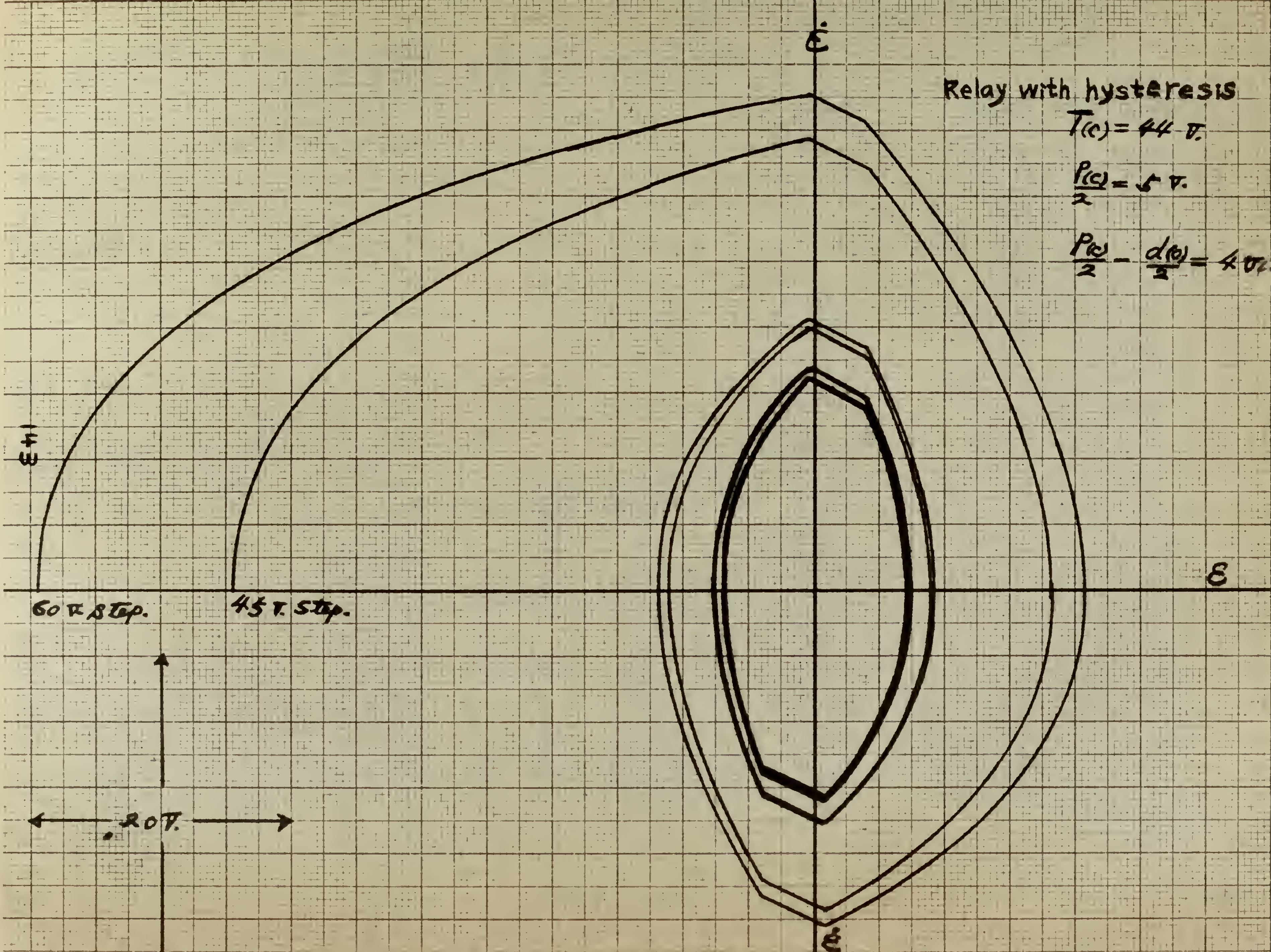

$\bar{T}(o) = 44\,v$
$\frac{P(o)}{2} = 5\,v$
$a \to \frac{P(o)}{2} - \frac{d(o)}{2} = 0$
$b \to \frac{P(o)}{2} - \frac{d(o)}{2} = 2\,v$
$c \to \frac{P(o)}{2} - \frac{d(o)}{2} = 4\,v$
$\dot{\varepsilon}$
ε
Relay with hysteresis
60v step
20v
a
b
c

Fig. 8-6



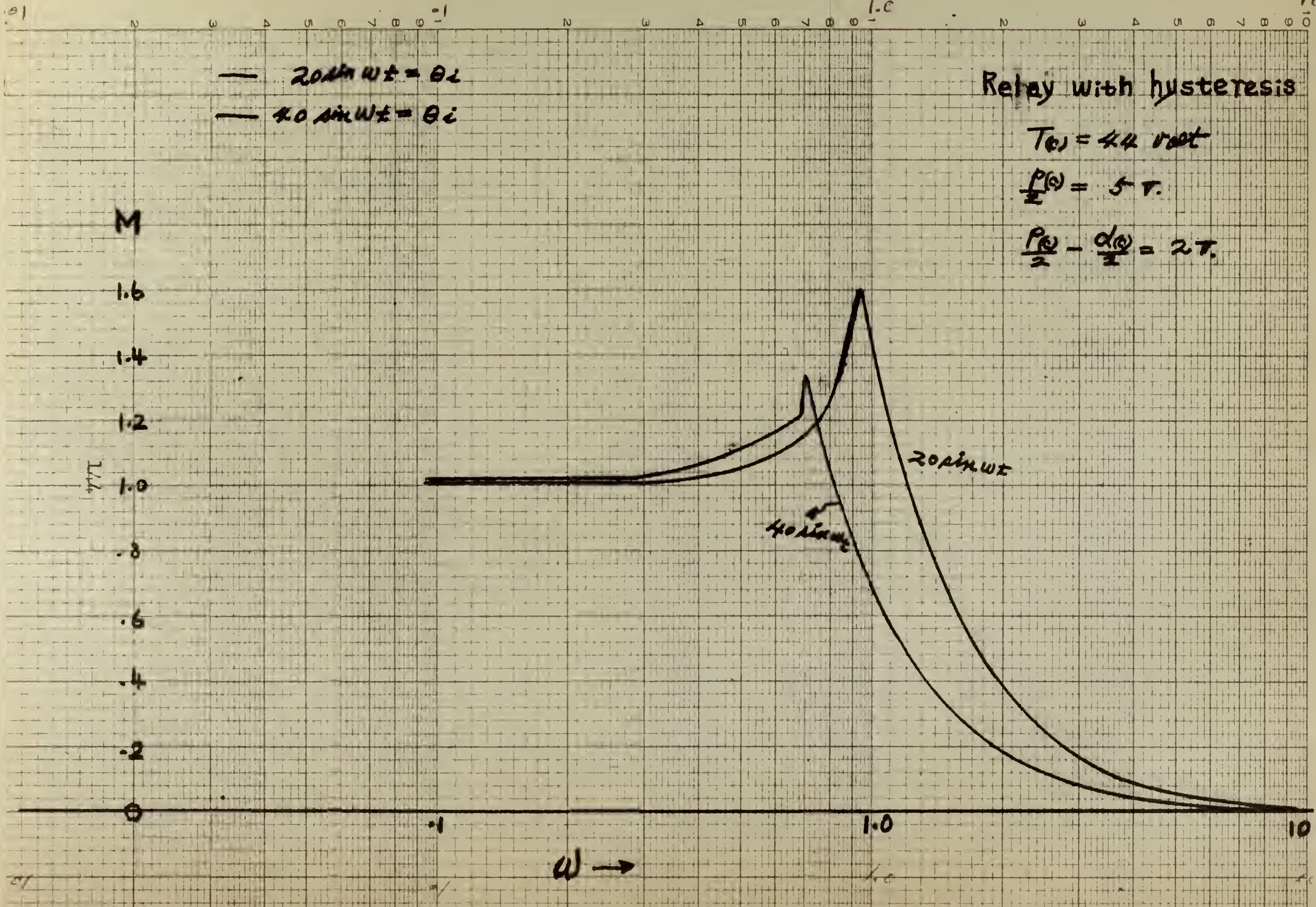
Relay with hysteresis
$T(c) = 44$ V.
$\frac{P(c)}{2} = 5$ V.
$\frac{P(c)}{2} - \frac{d(c)}{2} = 2$ V.
$\dot{\varepsilon}$
ε
60 V. Step.
45 V. Step.
20 V.

Fig. 8-7



Relay with hysteresis
$T(c) = 44$ V.
$\frac{P(c)}{2} = 5$ V.
$\frac{P(c)}{2} - \frac{d(c)}{2} = 4$ V.
$\dot{E}$
E
60 V. step.
45 V. step.
.20 V.

Fig. 8-8



20 sin ωt = θi
40 sin ωt = θi
Relay with hysteresis
T(s) = 44 volt
P(s)/2 = 5 v.
P(s)/2 − d(s)/2 = 2 v.
M
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
.1
1.0
10
ω →
20 sin ωt
40 sin ωt

Fig. 8-9

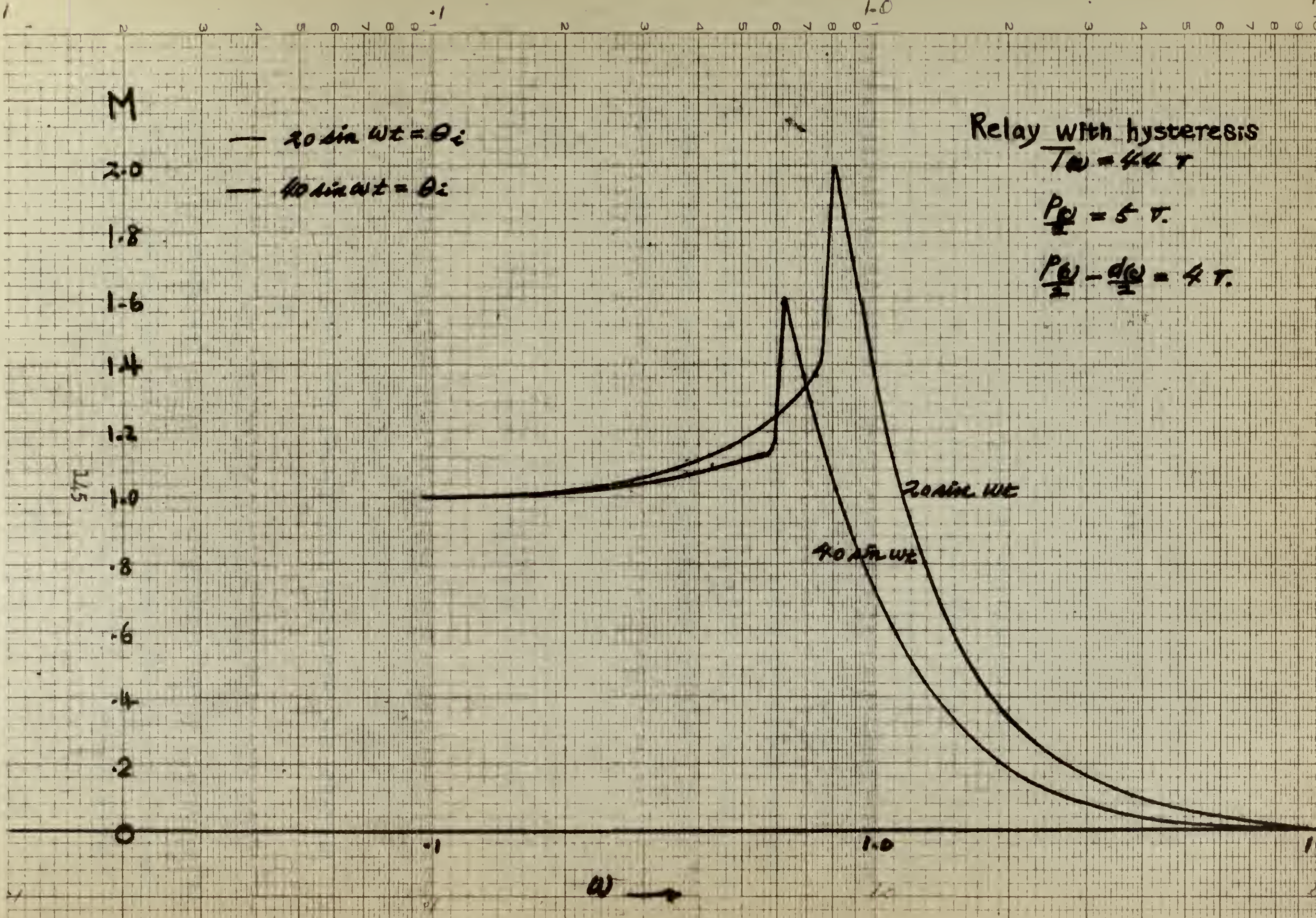

M
— 20 sin ωt = θi
— 40 sin ωt = θi
Relay with hysteresis
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
.1
1.0
10
20 sin ωt
40 sin ωt
ω

Fig. 8-10

4. Conclusion.

Simulation technique of basic nonlinearity encountered in usual servomechanism is generalized and analyzed in detail, with illustrative sample simulation of second order nonlinear system. Utilization of this technique is apparently easier and time saving with reasonable accuracy for most engineering purposes. The scaling technique was illustrated. The error in general is less than 5% more added to the error involved in the simulation of linear system, except cases of relay with dead zone and hysteresis and simple backlash. But those exceptional cases can be made to have errors within the prementioned value.

In all cases, error can be reduced by using more ideal components in simulation such as batteries, idealized diodes with more complicated setup or non-ideal diodes, or potentiometers well balanced. It can also be minimized by more fortuitous choice of scaling factors as was discussed in each paragraph of section 3, such as making scaling factor of the time greater so that an error resulting from noncritical component of the circuit may become negligible in terms of the physical system values.

The use of this technique in the study of the behavior of nonlinearity in servomechanisms saves a great deal of time and labor especially if one deals with the transient performance and frequency response of a system.

Given numbers for equations describing a physical system and numbers concerning nonlinearities, it is easier to set up circuit in the laboratory and simulate them to study or predict the system performance than drawing phase plane plots and tabulating data to further obtain

the transient response. When it comes to frequency response, the only mathematical approach "describing function method" does not give better accuracy, if as good.

For reasons stated above, the writer believes that the simulation method in the study of nonlinear behavior is preferable to any other method, and this method concerning basic nonlinearities commonly met are studied and analyzed with illustrative scaling in this report.